Frontiers in Health Informatics
ISSN-Online: 2676-7104
www.healthinformaticsjournal.com
2024; Vol 13: Issue 3
Open Access

Determination of Distribution Function for Large-Scale Wireless Communication Network

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Cite this paper as: Emmanuel Kwame Mensah,Mukil Alagirisamy,Eyram Kwame,Divya Midhunchakkaravarthy (2024) Determination of Distribution Function for Large-Scale Wireless Communication Network. *Frontiers in Health Informatics, 13* (3), 10966-10982

Abstract

This paper deals with the determination of signal distribution which is stochastic in nature in terms of transmitted and received signals in a metropolitan base station. An experiment was set up using a DS-CDMA system to transmit and receive signals at various places at the base station in the metropolitan wireless communication system. Both transmitted and received signals were recorded. The paper analyses their mathematical modeling, relationship, and their statistics based on large-scale measurement data from a metropolitan wireless network. The result indicates that the distribution of signal at the base station in terms of transmission is alpha-stable Weibull probability density function whereas the receive signals are Weibull extreme value distribution because the maximum log-likelihood estimate from Matlab 2021a is -1553.5 and -1147.87 respectively. Since the pdf of alpha-stable distribution is not analytically expressible except for certain parameter values, the researchers mathematically modelled the alpha-stable distribution in which the pdf exists $\alpha = 1.96$. *The theoretically modelled alpha-stable pdf and that of the data are compared and are found to be the same. Hence, it is theoretically and practically recommended that this method should be used in any field of Mathematics, Communication, and any other field where it uses is demanded. It also recommended for further study to unearth how the characteristic function transforms to the modelled pdf.*

Keywords

Probability density function (PDF), Moment Generating Function (MGF), Characteristic Function (CF), Maximum Likelihood Estimate (MLE),

Introduction

The determination of the distribution function for large-scale wireless communication networks is a crucial aspect of understanding and analysing the behaviour and performance of such networks. The distribution function provides insights into the statistical properties of various network parameters, such as signal strength, interference, and capacity, which are essential for designing efficient communication protocols and optimizing Network performance.

In wireless communication networks, the distribution function refers to the probability distribution that describes the likelihood of a particular event or parameter taking on different values. It characterizes the random nature of wireless channels and captures the variability in signal propagation, interference levels, and other factors that affect network performance[1-2]

One common approach to determining the distribution function for large-scale wireless communication networks is through empirical measurements. Researchers collect data from real-world deployments or conduct extensive field experiments to capture the statistical behaviour of network parameters. These tests may include signal strength measures at various sites, interference levels at various frequencies, and throughput measurements under varied network conditions[3-4].

By analysing these empirical measurements, researchers can estimate the distribution function using statistical techniques such as maximum likelihood estimation or kernel density estimation. These methods allow them to model the observed data and infer the underlying probability distribution that best fits the collected samples. The resulting distribution function can then be used to make predictions about network performance in different scenarios or evaluate the efficacy of various communication protocols [2].

Another approach to determining the distribution function for large-scale wireless communication networks is through theoretical modeling. In this approach, researchers develop mathematical models that capture the key characteristics of wireless channels and network behaviour. These models can be based on stochastic processes, such as random walk models or Markov chains, which provide a framework for describing the random variations in network parameters.

Theoretical models often make simplifying assumptions to facilitate analysis and computation. For example, researchers may assume that wireless channel fading follows a specific probability distribution, such as Rayleigh or Nakagami-m distributions, which are commonly used to model multipath fading in wireless channels. By incorporating these assumptions into the model, researchers can derive analytical expressions for the distribution function of network parameters [2, 5].

In addition to empirical measurements and theoretical modeling, simulation-based approaches are also widely used to determine the distribution function for large-scale wireless communication networks. Researchers develop network simulators that replicate the behaviour of real-world wireless networks and generate synthetic data based on predefined network models and parameters. By running simulations with different input parameters and scenarios, researchers can collect a large amount of data and estimate the distribution function from the simulated results.

Simulation-based approaches offer flexibility in studying various network configurations and scenarios that may be challenging or costly to replicate in real-world experiments. They also allow researchers to control and manipulate different network parameters to understand their impact on the distribution function. However, it is crucial to ensure that the simulation models accurately capture the essential characteristics of real-world wireless networks to obtain reliable results [6].

In summary, determining the distribution function for large-scale wireless communication networks involves a combination of empirical measurements, theoretical modeling, and simulation-based approaches [7]. These methods offer vital insights regarding statistical properties of network parameters and enable network designers to design efficient communication protocols, optimize network performance, and evaluate system-level performance metrics.

Literature Review

Probability Density Function of Large-scale Wireless Communication Networks

This section explains the definitions and results of random variables and random vectors in a random process. A random process describes a physical quantity based on a parameter, which can be time or coordinate. A random variable is a function or mapping defined on a probability space, which is measurable and consists of sample space, event space, and probability. In communication, random variables like interference and noise can be additive or multiplicative. Large-scale fading, or shadowing, is caused by radio wave absorption and changes in propagation media. The cause of fast fading includes changing phase conditions on the propagation path, causing significant variations in effective magnitude and phase [6-7]. According to [7], the most important characteristics of a random variable are the probability density function and characteristic function which play a critical role in wireless communication networks.

Probability Distribution

A probability distribution is a non-negative set function that maps elements in a sample space to a real number line bounded between zero and one inclusive.

Cumulative Distribution Function (CDF)

In probability and statistical theory, CDF describes the probability distribution of a random variable given as

$$F_X(x) = \Pr(X < x) \quad \text{.................................1}$$

Probability Density Function (PDF)

The probability density function, $f_X(x)$ describes a continuous random variable and is mathematically defined as

$$f_X(x) = \frac{d}{dx}\left(F_X(x)\right) \quad \text{..........................2}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(s)ds \quad \text{............................3}$$

Characteristic Function

The characteristic function (CF), $\Theta_X(jv)$ is defined as the Fourier transform of the corresponding pdf of a continuous random variable:

$$\Theta_X(jv) = \int_{-\infty}^{\infty} f_X(x)e^{jvx}dx \quad \text{..................................4}$$

Moment Function

Moment functions, $M_{i_1}(t), M_{i_1 i_2}(t_1, t_2), \ldots,$ of a random variable $X(t)$ in the random process is defined as follows. Let $I = [i_1, i_2, \ldots, i_n]$ be a set of integers greater than or equal to zero, then for different moments at time, $t_1, t_2, \ldots, t_n$ can define a $n-$dimensional moment function of order $\alpha = \sum_{i=1}^{n} i_k$ as

$$M_{i_1 i_2 \ldots i_n}(t_1, t_2, \ldots, t_n) = \left\langle X^{i_i}(t) X^{i_2}(t) \ldots X^{i_n}(t) \right\rangle$$
$$= \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} x_1^{i_1} x_2^{i_2} \ldots x_n^{i_n} f_{M\xi}(x_1, x_2, \ldots, x_M; t_1, t_2, \ldots, t_M) dx_1 dx_2 \ldots dx_M \ldots\ldots\ldots\ldots 5$$

Moment Generating Function

The moment-generating function (mgf) is defined as the Laplace transform of the probability density function. It is mathematically defined as

$$M_X(s) = E(e^{sX}) = \int_{-\infty}^{\infty} e^{sx} f(x) dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

It serves as a bridge between the distribution X and its moments

Cumulant

Cumulants serve as an alternative to probability distribution moments. They are defined by taking the cumulant-generating function, which is the natural logarithm of the moment-generating function. The cumulants offer a different viewpoint on the properties of a distribution when matched with the moment.

$$K_{X_1+X_2+\ldots+X_m}(s) = In\left(E(e^{s(X_1+X_2+\ldots+X_m)})\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

Conceptual Framework

The epistemology (Knowledge) of covariance function allows the researcher to derive characteristic functions and pdf. This is because PDF and CF are the two functions needed to completely characterize interference and other signals in communication. This implies that an adequate description of the random process can be accomplished by a series of PDFs, CF, moments, or cumulant as indicated in Figure 1.

Figure 1 is a conceptual framework, based on the principles of probability that derives this research and it fit the problem under discussion. Below are statements based on the mathematical underpinnings:

I. If by applying the Laplace transform of the pdf, moments of the function are derived, then by inverse Laplace transform on the moment, pdf will be derived back.

II. An importantant difference between the Taylor series and the asymptotic series is that the Taylor series must *converge* if they are to be useful.

III. Based on the conceptual model, there are four possible gates to the model. This describes four ways under which the problems can be handled.

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It is crucial to note that the decision about which characteristics of a random process are used heavily depends on the type of random process. For Gaussian process which is completely characterised by its mean and covariance function can be applied with the same complexity.

In the future, theoretically, the researcher is proposing that the same problem should be looked at using either gates 2,3, or 4 of which the major steps are clear in Figure 1. For example, from gate 4, the first two cumulant functions $M_1(t)$ and $K_2(t_1, t_2)$ play a central role in the analysis of random processes and their transformation by linear and nonlinear systems. The correlation theory of random processes is a subset of random process theory that focuses on the first two cumulant functions.

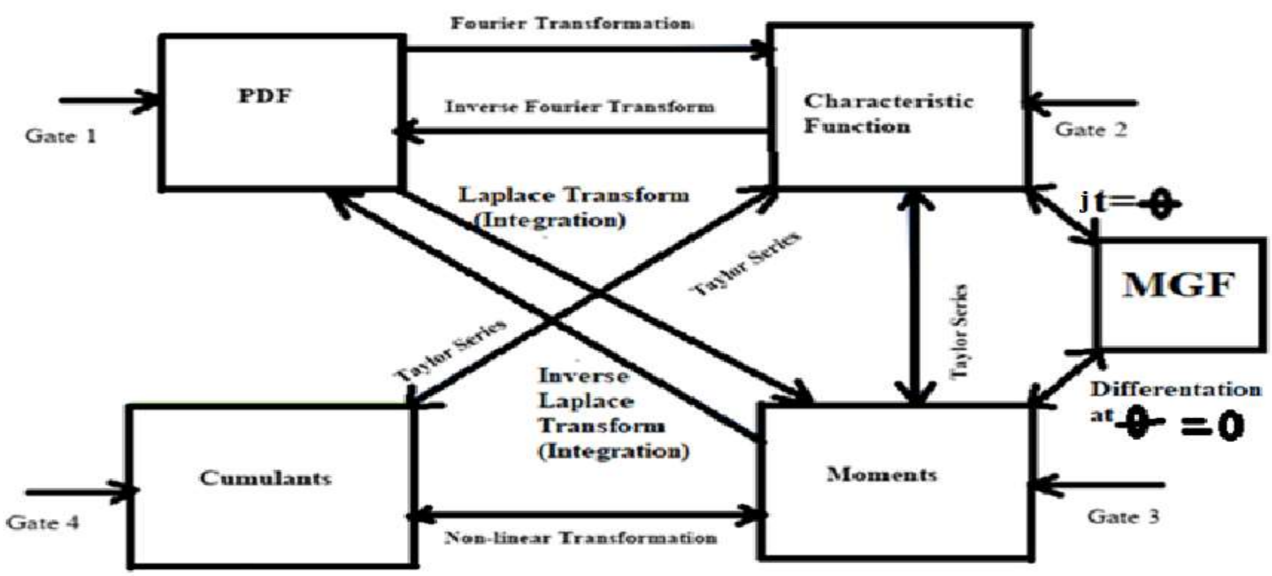

Figure 1: Block Diagram of Theoretical links between PDF, CF, moments and cumulants

In gate 3, the principles of mathematical modeling based on probability and stochastic theories shall be applied to deduce the moments of the networks. After which either the Taylor series will be applied to derive the characteristic function of the network then the inverse Fourier transform applied to get the PDF of the wireless network or a series of differentiations to derive the PDF then the Fourier transform applied on the result to derive the characteristic function of the wireless networks.

Gate two will start with numerical methods and analysis to derive the characteristic function from which the researcher either applies the Taylor series to derive moments or inverse Fourier transform to find the PDF of the wireless network.

Finally, in gate one there are two application possibilities namely spherical invariant random processes (SIRP) theory and stochastic geometry theory (SG) to derive PDF of wireless communication networks. At this point, the researcher is committed to solving the problem using SG [8-10].

Stable Distribution

The general characteristic function can be stated analytically, but the pdf for a general stable distribution cannot [11]. Theoretically, pdf is defined by the inverse Fourier transform of a characteristic function $\Phi(t)$ is given by

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi(t)e^{-jxt}dt.......................34$$

where

$$\varphi(t;\alpha,\beta,\gamma,\delta)=\begin{cases}\exp\left(jt\delta-|\gamma t|^{\alpha}-j\beta\,\mathrm{sgn}(t)\tan\left(\frac{\pi\alpha}{2}\right)\right), & \alpha\neq 1\\ \exp\left(jt\delta-|\gamma t|^{\alpha}-j\beta\,\mathrm{sgn}(t)\frac{2}{\pi}In(t)\right), & \alpha=1\end{cases}........35$$

where

$$sign(t)=\begin{cases}-1, & t<0\\ 0, & t=0\\ 1, & t>0\end{cases}$$

The stable distribution uses the parameters listed in Table 1.

Table 1: Stable probability distribution parameters

Parameter	Description	Support
alpha (α)	First shape parameter Characteristic Exponent or Stability index, Tail Index, Tail Exponent	$\alpha \in [0,2]$
beta (β)	Second shape parameter Skewness parameter	$\beta \in [-1,1]$ $\beta=0$ Symmetrical distribution $\beta>0$ Right-skewed distribution $\beta<0$ Left-skewed distribution
gamma (γ)	Shape (Scale) parameter	$\gamma \in]0,\infty[$
delta (δ)	Location parameter or shift	$\delta \in]-\infty,\infty[$

	parameter	

The shape parameters of a distribution, such as α, β, scale, and location, play a crucial role in determining the probability distribution of signals. The smaller α, the greater the frequency and size of the extreme event. When α approaches 1, the distribution has a mean. As discussed in [11], there are arithmetic issues in reliably estimating the pdf and CDF when the parameter is near 1 or 0. If α is near to 1, the software rounds to 1. If α is near to zero, the densities may be inaccurate. When α is between 0 and 2, the distribution is impulsive. The second shape parameter, β, describes the distribution's skewness.

Stable distributions are typically expressed as $X \sim S(x:\alpha,\beta,\gamma,\delta)$. However, the only known closed-form PDF and CDF expressions for stable distributions are Gaussian, Cauchy, and Levy. This is an important problem in fields such as communication and finance to unearth other distributions to help further work [9-10].

Extreme value analysis (EVA) is a field of statistics that deals with severe deviations from the median of probability distributions. It aims to assess the probability of events that are more severe than any previously seen. The Theory of Extreme Value (EVT) gives the statistical analysis for studying the distributions of extreme order statistics of collections of random variables. The independent and identical (i.i.d) $S\alpha S$ random variables driving the extreme distribution exhibit the following characteristics: limit distribution, Asymptotic distribution, and Weak convergence. It has applications in communication theory and signal processing, particularly in the estimation via extrapolation of very small probabilities involved in the assessment of communication device performance and signal processing algorithms

Methodology

Experimental Setup

The DS-CDMA communication technique is based on chaotic dynamics, and the equipment needed to gather data in wireless communication is depicted in the block diagram Figure 2 below. The system is divided into three sections: signal generator and recorder, transmitter, synchronization, and receiver.

A signal is generated, recorded, sent, and received. Additive white Gaussian noise is added to the channel. The transmitted signal with the noise is synchronized, received, and recorded at the receiver.

Mathematical Model

The pdf of a modified Weibull distribution is defined as follows

$$f_1(x) = \exp\left(\frac{x-(\mu_\omega + a_\omega)}{\alpha a} - \frac{1}{b\alpha}\exp\left(\frac{x-(\mu_\omega + a_\omega)}{\alpha c}\right)\right) \text{......................} 36a$$

here a,b,c and a_w are constants with mean μ_ω. The a_w is the translational vector. This distribution is a stable function due to the presence of stable parameter α in the model. The pdf is Generalised Extreme Value Distribution (GEVD) of type III is a Weibull distribution is given as

$$f_2(x) = \frac{1}{\sigma}\left(\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}\right)^{\xi+1} \exp\left(-\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}\right) \text{.....................} 36b$$

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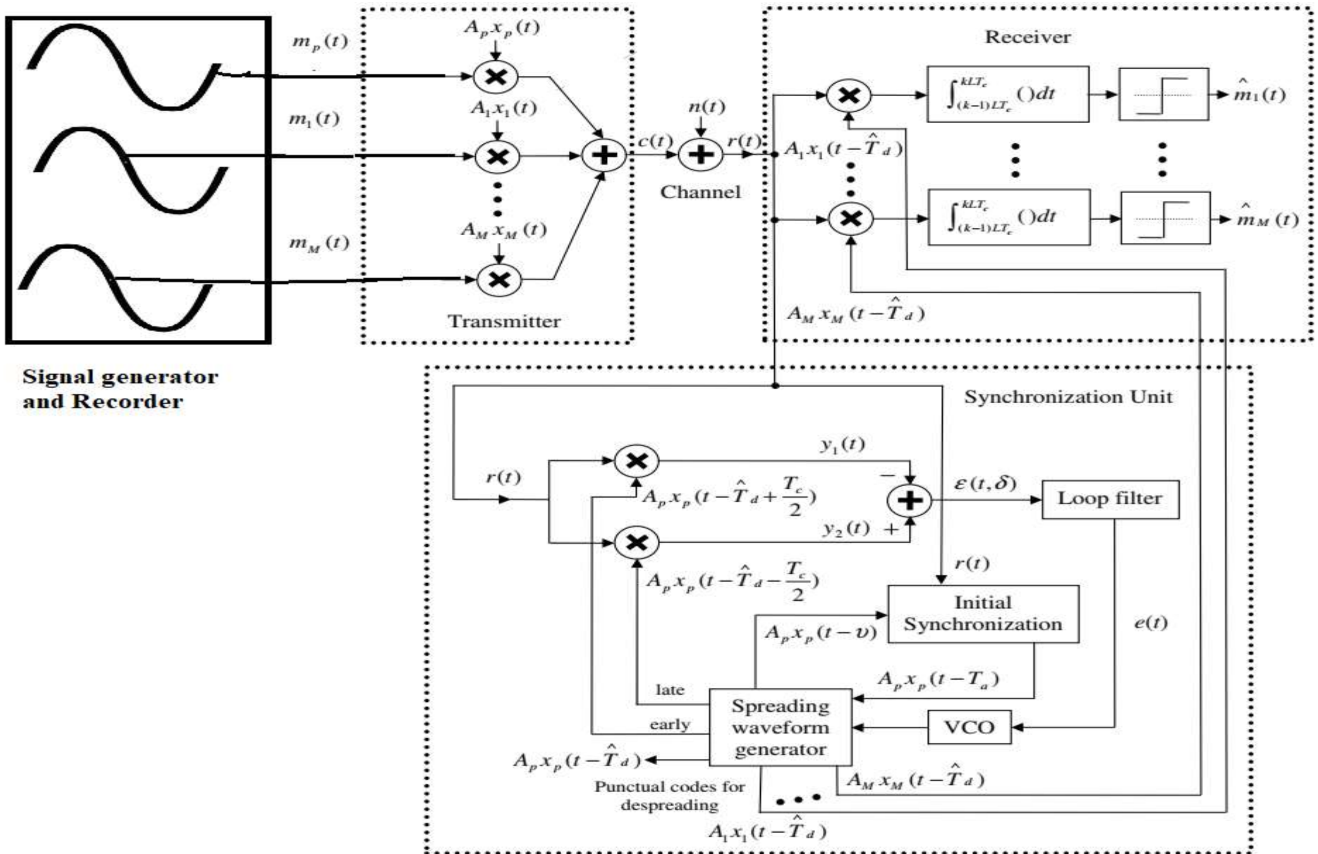

Figure 2: DS-CDMA chaotic communication system with synchronization unit

Theory of Maximum Likelihood Estimation (MLE)

This is an estimation method used to calculate parameters of a population based on probability theory used to determine which probability distribution best fits or describes the statistical data of distribution by applying likelihood estimate, AIC, and BIC values. From the Likelihood estimate, we choose a probability distribution that has a maximum likelihood evaluation among a set of probability distributions. Furthermore, from the AIC and BIC point of view, it seeks to fit a probability distribution that has the best minimum error. It is interesting to note that the maximum likelihood evaluation always coincides with the best minimum error estimate from AIC and BIC. Based on this result the best-fitted probability distribution is chosen.

The MLE is mathematically given by

$$\hat{\theta} = \arg \max_{\theta \in \Theta} L_n(\theta; x) \qquad \text{...36}c$$

where $\hat{\theta} = \hat{\theta}_n(x) \in \Theta \quad and \quad \hat{\theta}_n : \mathbb{R}^n \to \Theta$ is measurable. It is a function defined over a sample space which has the attributes of stochastic. The geometry of the sample space makes it stochastic geometry. Hence this justifies the application of MLE is Stochastic Geometry. Moreover, a sufficient but necessary

Frontiers in Health Informatics
ISSN-Online: 2676-7104
2024; Vol 13: Issue 3

condition for its existence is for likelihood function to be continuous over a parameter space Θ that is compact which means closed and bounded sets. This closed and bounded nature also defines the possibility of applying control theory to manipulate the situation to its best conclusion in order to un-earth hidden fact unknown to the world. Another interesting fact about this MLE is the using of best maximum estimate which to the researcher is another word for optimal. Hence a possibility of applying optimal control theory. This justifies possibility of application of stochastic optimal control in Large-scale wireless communication networks[10-11].

Numerical Results

Transmitted Power Distribution at the Base station

The experimented data for transmission in the metropolis was analysed and plotted at 95% confidence interval is illustrated is Figure 3.

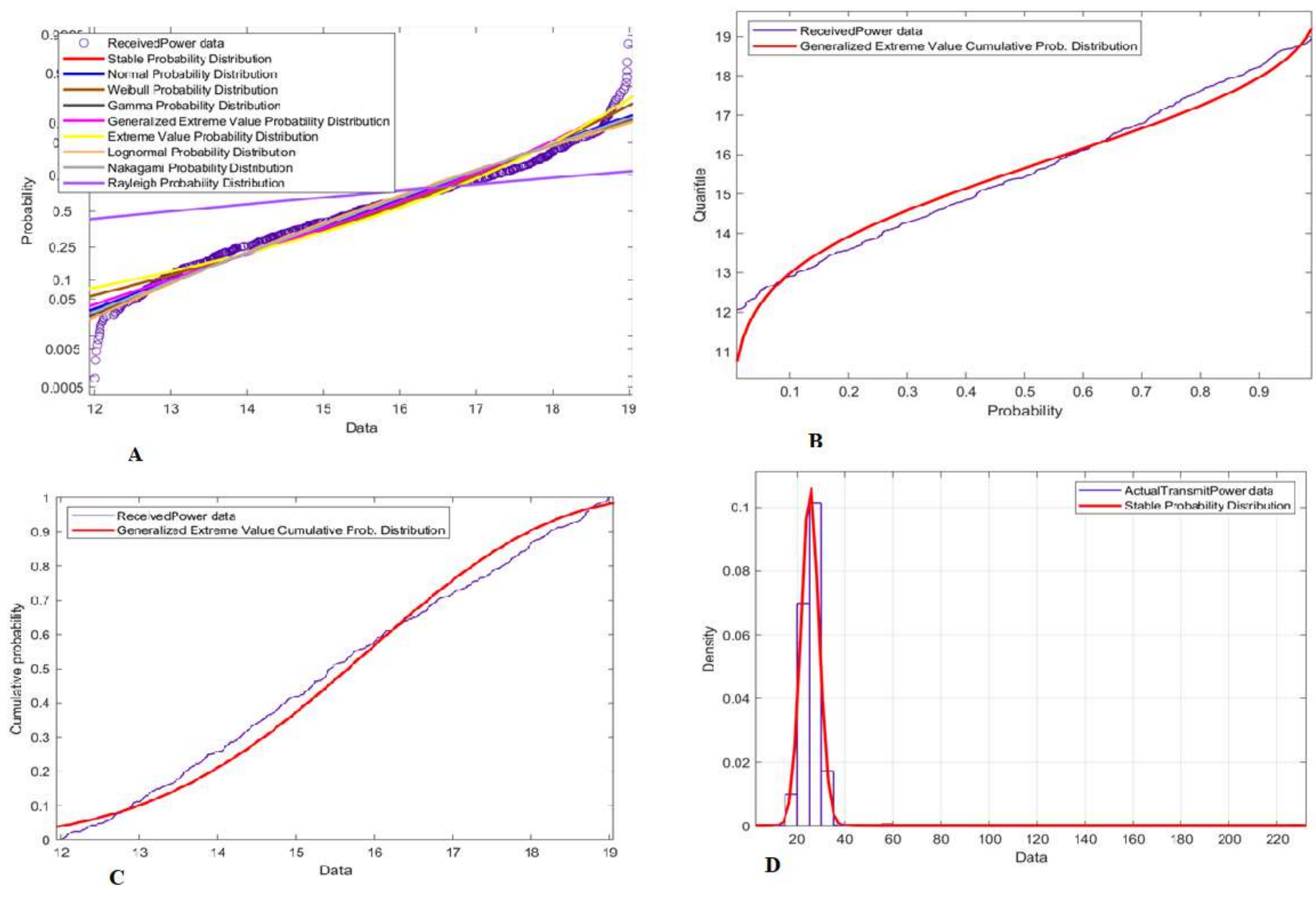

Figure 3: Graphs at transmission station

Figure 3 A, B, C, and D are probability, quantile, cdf, and pdf plots respectively at the transmission station. Figure 3D illustrates the histogram of the transmitted signal with the alpha-stable distribution function whereas C is the cumulative graph of the transmitted signal.

From Figure 3.0, the best-fitted probability distribution function at the transmitting base stations within the metropolitan area is alpha stable probability distribution function at a 95% confidence interval with the following statistics illustrated in Table 2.

Table 2: Statistics of the distribution at transmission base station.

Distribution	Alpha Stable	
Log likelihood	-1553.5	
Domain	$y \in]-\infty, \infty[$	
Mean	25.3886	
Variance	NaN	
Parameter	Estimation	Standard Error
Alpha	1.95756	0.0346363
Beta	0.315347	0.67466
Gam	2.63126	0.087114
Delta	25.3332	0.178758

Estimated Covariance				
	Alpha	Beta	Gam	Delta
Alpha	1.19967×10^{-3}	3.19855×10^{-3}	8.75751×10^{-4}	6.16233×10^{-4}
Beta	3.19855×10^{-3}	4.55166×10^{-1}	3.90443×10^{-3}	-5.02424×10^{-2}
Gam	8.75751×10^{-4}	3.90443×10^{-3}	7.58886×10^{-3}	5.215×10^{-4}
Delta	6.16233×10^{-4}	-5.02424×10^{-2}	5.215×10^{-4}	3.19543×10^{-2}

From characterization theorem of probability theory and mathematics statistics of maximum log likelihood estimation combined with graphical theory, the transmitted signals of the base station is Alpha-stable distributed at 95% confident interval. From Table 2, the parameters of the alpha stable distribution are $\alpha = 1.96, \beta = 0.32, \gamma = 2.63, \text{ and } \delta = 25.33$ with 2 decimal places accuracy. The characteristic function of stable distribution is given as

$$\varphi(t;\alpha = 1.96, \beta = 0.32, \gamma = 2.63, \delta = 25.33) = \exp\left(25.33\,jt - 2.63|t|^{1.96} - 0.32\,j\,\mathrm{sgn}(t)\tan\left(\frac{1.96\pi}{2}\right)\right) \quad \text{.......37}$$

$$sign(t) = \begin{cases} -1, & t < 0 \\ 0, & t = 0 \\ 1, & t > 0 \end{cases}$$

Replacing jt by θ and simplifying, we have the MGF of stable distribution as

$$M(\theta) = \exp\left(25.33\theta - 2.63\left|-j\theta\right|^{1.96} - 0.32\,\mathrm{sgn}(\theta)\tan\left(\frac{1.96\pi}{2}\right)\right) \text{..................38}$$

where

$$sign(\theta) = \begin{cases} -1, & \theta < 0 \\ 0, & \theta = 0 \\ 1, & \theta > 0 \end{cases}$$

$$at \quad 95\% \quad confidence \quad \mathrm{int}\, erval.$$

from equation 36a (modified Weibull distribution) is given by

$$f_1(x) = \exp\left(\frac{x-(\mu_\omega + a_\omega)}{a\alpha} - \frac{1}{b\alpha}\exp\left(\frac{x-(\mu_\omega + a_\omega)}{c\alpha}\right)\right)\Bigg|_{\substack{\alpha=1.96 \\ \mu_\omega = 25.3886}}$$

$$= \exp\left(\frac{x-(25.3886 + a_\omega)}{1.96a} - \frac{1}{1.96b}\exp\left(\frac{x-(25.3886 + a_\omega)}{1.96c}\right)\right) \text{.............39}a$$

where a,b,c and a_ω are constant, μ_w is the mean and α is the standard alpha stability parameter for stable probability distribution functions.

Fiting the graph, $a = 1.35, b = 1.01, c = 2,$ and $a_\omega = 2.4014$. Hence

$$f_1(x) = \exp\left(\frac{x-(25.3886 + 2.4014)}{2.646} - \frac{1}{1.9796}\exp\left(\frac{x-(25.3886 + 2.4014)}{1.96c}\right)\right) \text{...........39}b$$

The graph of equation 39b is given in figure 4 below:

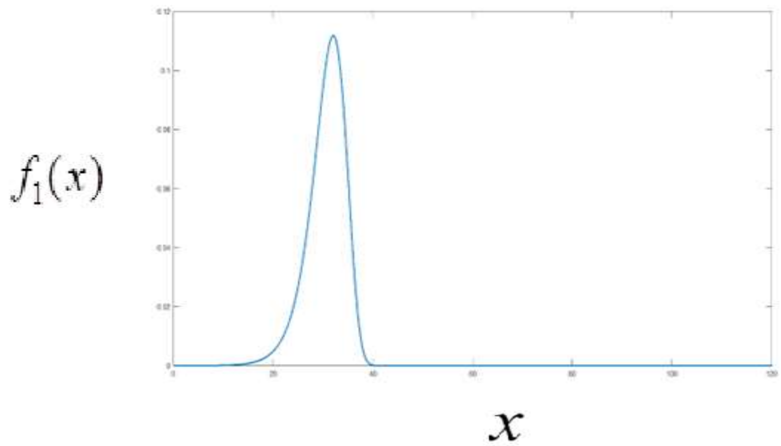

Figure 4: Density graph of $f_1(x)$.

Figure 4 is the probability density function graph of $f_1(x)$. Comparing Figure 4 with figure 3D, in both graphs, the stable distribution is non-Gaussian in nature and skewed to the right. Moreover, near the origin, both behave like a Gaussian density function, but as it moves away from the origin, it loses its Gaussianity. Another crucial aspect of the function's behaviour seen in the graph is that it asymptotically approaches the horizontal axis as more time passes. This behaviour is consistent with the modest convergence of the signals. Therefore, the asymptotic weak convergence theorem is required for the characterization of large-scale wireless communication networks. It implies existing limit distribution functions and normalizing constants from a theoretical standpoint. The researchers concluded that they have the same characteristics. Hence $f_1(x)$ is the pdf at the transmission base station.

From the conceptual framework model, another analytical confirmatory test which hold is

Characteristic Function, CF= $\frac{1}{2\pi}\int_0^\infty f_1(x)e^{jtx} = F_1(t) = \phi(t)$, and

pdf , $$f_1(x) = \frac{1}{2\pi}\int_0^\infty F_1(t)e^{-jtx}dt$$

This discovery method serves as the foundation for further study by mathematically finding out under what condition does $F_1(x)$ transforms to $f_1(x)$. Moreover, it also serves as a contribution to the field of academia and research. In a predictive implication, the researchers are 95% confidently sure that the optimal detection in large -scale wireless communication will be asymptotic since every aspect of data transmission and reception is affected by interference.

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ISSN-Online: 2676-7104
2024; Vol 13: Issue 3
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Received power distribution at the base station

At the receiver section of the base station within the metropolis, the graphs of the experimental data are illustrated in Figure 5.

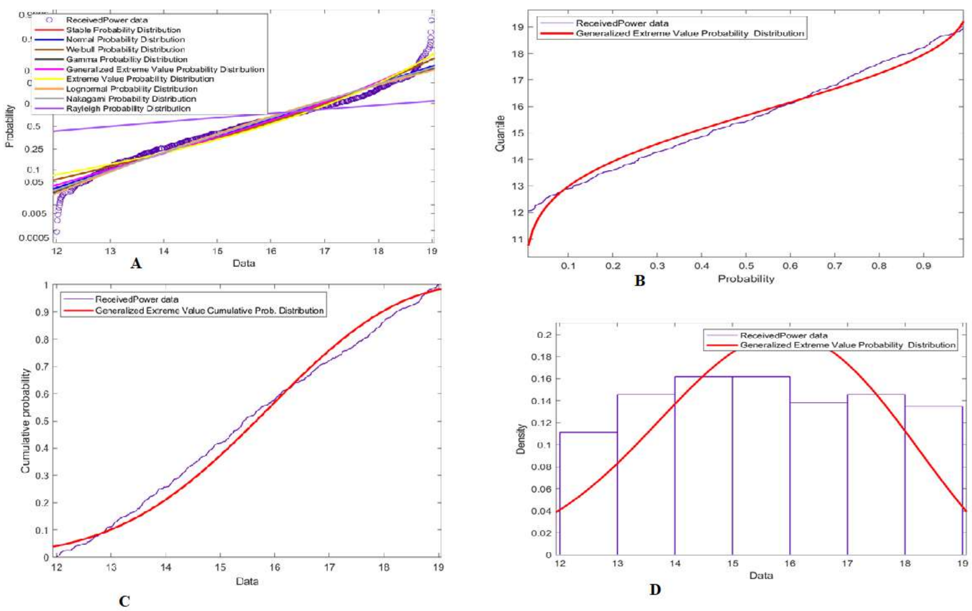

Figure 5: Graphs of experimental data.

Figure 4 A, B,C, and D are probability, quantile, cdf, and pdf plots respectively at the receiver station.

Figure 4D illustrates the histogram of the received signal with the generalized extreme value probability distribution function whereas C is the cumulative graph of the received signal. Figure 4A illustrates the probability plot of the various distribution to find which distribution best fits the data at the receiver station.

From the characterization theorem of probability theory and mathematics statistics of maximum log-likelihood estimation combined with graphical theory, the receiver signals of the base station within the Metropolitan area is generalized extreme value distributed at 95% confident interval with the following statistics illustrated in Table 3.

Table 3: Statistics of the distribution at receiver base station.

Distribution	Generalized Extreme Value Probability Distribution
Log likelihood	-1147.87
Domain	$y \in]-\infty, \infty[$
Mean	15.54

Variance	3.6483	
Parameter	Estimation	Standard Error
Xi, ξ (Shape parameter)	-0.397183	0.0472014
Sigma, σ (Scale parameter)	2.0105	0.0815198
Mu, μ (Location parameter)	14.9701	0.100732

Estimated Covariance			
	ξ	σ	μ
ξ	2.22797×10^{-3}	-2.83799×10^{-3}	-2.45869×10^{-3}
σ	-2.83799×10^{-3}	6.64548×10^{-3}	7.99521×10^{-4}
μ	-2.45869×10^{-3}	7.99521×10^{-4}	1.01469×10^{-2}

From Table 3, the receiver power distribution is generalized extreme value probability distribution (GEVD) with $\xi = -0.40, \sigma = 2.01 \quad and \quad \mu = 14.97$ at 2 decimal places. Further, GEVD can be distinguish into three types due to its shape parameter. From statistical theory whenever the shape parameter is less than zero, the distribution is Weibull extreme value distribution. This distribution has the property of both Weibull and extreme value distributions and it is heavy tailed in nature.

Probabilistic nature of the extreme value theory deals with the stochastic behaviour of the maximum and the minimum of independent and identical (i.i.d) random variables. The underlying distribution of the upper and lower tails affect the distributional features of extremes (maximum and lowest), extreme and intermediate order statistics, and exceeding high or low thresholds.

Statistical procedures can analyse the tail of a distribution function or its functional parameters using extreme and intermediate order statistics or exceeding high thresholds. By focusing on the tails, we may propose personalised statistical models.

The term "extreme value" is attached to these distributions (Gumbel, Fréchet, and Weibull) because they can be obtained as limiting distributions (as $n \to \infty$) of the greatest value among n independent random variables each having the same continuous distribution. By replacing X by $-X$, limiting distributions of least values are obtained.

Although the distributions are known as extreme value, it is to be borne in mind that they do not represent distributions of all kinds of extreme values (e.g., in samples of finite size), and can be used empirically (without an extreme value model).

Mathematically to characterized as distribution is to state it's probability density function and moment generating function or characteristic function. Since the shape parameter $\xi = -0.4$, from equation 36b

$$f_2(x)=\frac{1}{\sigma}\left(\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}\right)^{\xi+1}\exp\left(-\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}\right)\Bigg|_{\substack{\mu_\omega=15.54\\ \xi=-0.4\\ \sigma_\omega=2.01}}.....................40a$$

$$=\frac{1}{2.01}\left(1-0.4\frac{(x-15.54)}{2.01}\right)^{1.5}\exp\left(-\left(1-0.4\frac{(x-15.54)}{2.01}\right)^{2.5}\right)................40b$$

The graphical fit for equation 40 is illustrated in Figure 6.

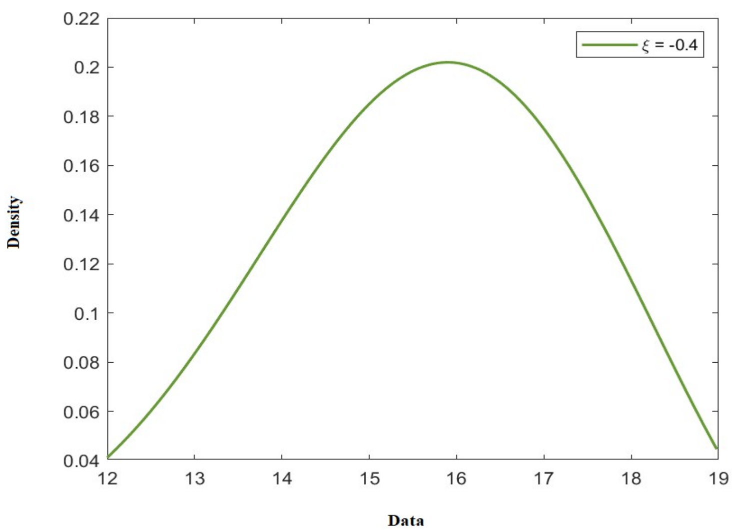

Figure 6: Graph of pdf at the receiver base station

From Figure 5, comparing it with figure 4D exhibit the same characteristics. This confirms equation 40 to be the real pdf at the received base stations which is GEVD type III which is a Weibull extreme value distribution.

The CDF distribution is the GECD type III is Weibull distribution for the received signal is given as,

$$M(\theta)=e^{14.97\theta}\left[e^{\frac{2.01\theta}{0.4}}\left(\sum_{r=0}^{\infty}\frac{(\theta r)^{r+1}}{r!(-0.4)^r}\right)\sum_{n=0}^{r}(-1)^n\ {}^nC_r\frac{r(1+0.4(n+1))}{-0.4(n+1)}\right]...........41a$$

$$=e^{14.97\theta}\left[e^{5.025\theta}\left(\sum_{r=0}^{\infty}\frac{(\theta r)^{r+1}}{r!(-0.4)^r}\right)\sum_{n=0}^{r}(-1)^n\ {}^nC_r\frac{r(1+0.4(n+1))}{-0.4(n+1)}\right]........41b$$

This describes completely characterized distribution in wireless communication. This because the characteristic function, moments and cumulant can be easily be derived from the MGF. For example, replacing θ with it describes the CF of GEVD. Hence, the deduction of the CF and pdf are analytically true and given as follows:

$$CF, \quad F_2(t) = \frac{1}{2\pi}\int_0^\infty f_2(x)e^{jtx}dx$$

and

$$pdf, \quad f_2(x) = \frac{1}{2\pi}\int_0^\infty F_2(x)e^{-jtx}dt$$

Mathematically, the properties that characterized Weibull Extreme Value Distribution are as follows:

I. Asymptotic in nature.
II. Either short- or long-range dependence.
III. Subexponentially exponential distribution
IV. Moments may or may-not exist.
V. Tail-index and are skewed.

The result form this study confirms that in large-scale wireless communication network, the distribution of power signals is characterized by tailed probability distribution which can be heavy, light or fat. The asymptotic behaviour of the power signal has an implication on the optimal power detection within the wireless network. This is because optimal power detection depends of the received power signal in the base station. In some of the heavy-tailed distributions, their moments do not exist, other exist with flower order moments which are fractional in nature example is an alpha-stable distribution where $\alpha \in \left]0,2\right]$. Furthermore, light-tailed distribution such as Gamma distribution has their moments existing with either long- or short-range dependence.

Conclusion and recommendations

Conclusion

We conclude at a 95% confidence interval that the transmitted signals at the base station is alpha-stable Weibull probability density function distribution whereas the receiver signals are Weibull extreme value distribution. Moreover, it is these distributions that describe the network signals in the large-scale wireless communication network in the metropolitan area. In addition, an alpha-stable parameter value of 1.96 is a heavy-tailed distribution, resulting in potential outliers and higher signal variability. This implies irregular behaviour, poor system reliability, and lower performance. To solve this, measures such as error correction codes, adaptive modulation and coding, diversity techniques, and precise channel estimation may be applied. Error correction codes identify and rectify faults caused by signal fading or interference, whilst adaptive modulation and coding improve data transmission efficiency. These interventions may be useful in the maintenance of reliable wireless communication in systems with significant tail distributions.

Recommendation

The study recommends that the determination of probability function of stable distribution should be looked at through the lenses of probability plots and numerical statistical modelling of Weibull distribution with curve fitting techniques.

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