An EPQ Model For Non-Instantaneous Deteriorating Items With Finite Production Rate And Time Dependent Demand Under The Effect Of Partial Backlogging With Credit Financing Policy

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ABSTRACT

This paper investigates the optimal replenishment policy for non-instantaneous deteriorating items under the policies of credit financing policy within the EPQ framework. Production rate is finite and shortages are allowed subject to partially backlogging. The backlogging rate varies inversely as the waiting time for the next replenishment. Moreover, the demand is considered to be time dependent demand which suits the real scenario of the business industry. The objective of this work is to minimize the total inventory cost and to find the optimal length of replenishment and the optimal order quantity. Computational algorithms for this model are designed to find the optimal order quantity and the optimal cycle time. The numerical examples are given and the sensitivity analysis of the parameters introduced are carried out to demonstrate the application and the performance of the proposed model.

KEYWORDS

Time dependent demand, credit financing policy, partial backlogging.

INTRODUCTION

Inventory control also called stock control, is the process of managing inventory levels, which is also related to many aspects of the production organization, can affect the business and play an important role in operations management activities. Early and classical models of economic production systems had many limiting assumptions. However, many of the initial assumptions were modified over time. As a result, more complex and extensive assumptions such as delayed payment, deteriorating goods, dynamic and variable demand, or discounts have been considered. Delayed payment is now a valuable promotional tool for manufacturers to increase their profits by further stimulating sales. It is an excellent opportunity for manufacturers or retailers to reduce demand uncertainty. In other words, when the manufacturer sends the ordered units to the retailer without payment, it transfers the responsibility of storage and its costs to the retailers while taking the risk of demand uncertainty. The manufacturer motivates his customers to trade and encourages them to place their orders in higher quantities. This method, known as trade credit, is used as an incentive policy to attract more customers and increase customer satisfaction.

The delayed payment time can be during the production cycle or outside the production cycle. In cases where the purchase cost has not been paid, capital expenditures are not considered for goods in the warehouse because there is no capital involved in the inventory. At the same time, the cost of capital is higher than the interest rate of sold goods for which the money has not yet been paid. Furthermore, one of the assumptions of classical inventory control models is that the demand rate is constant. However, demand may not be fixed in practice and may depend on time, price, inventory level, etc. and many goods are deteriorating and would deteriorate over time. Foods, medicine, and grains are some examples of deteriorating items. Hence considering the deterioration of things will give more accurate results. This paper discusses the above concerns and a production inventory model is developed considering time dependent demand, delayed payment policy for deteriorating products under the effect of partial backlogging.

LITERATURE REVIEW

Credit trading was studied for the first time by Haley & Higgins (1973). They considered the impact of a two-part trade credit policy on the optimal balance and payment policy. Two-part commercial credit refers to items in which the manufacturer considers a cash discount paid over a while and a specified period in a more considerable credit period. Chapman et al. (1984) will develop optimal replenishment policies under the delayed payment for Economic Order Quantity (EOQ) model. They considered the EOQ model with constant demand in which the shortage is not allowed (Chapman et al., 1984). Goyal (1985) examined the delayed payment in the EOQ system and assumed the manufacturer would allow the retailer to have a predetermined period for settling its order account. Then provide a mathematical model for determining the amount of economic order. Therefore, Goyal (1985) used credit purchasing in inventory control models as a mathematical model for the first time. Teng (2002) modified the Goyal (1985) model to assume the unit price and cost difference. They showed that the retailer should order smaller to take advantage of delayed payments and make more profit (Teng, 2002). Abad & Jaggi (2003) examined the seller-buyer inventory model in which the seller used trade credit, and the buyer used the EOQ model with no shortages. They formulated the seller-buyer relationship, considering the unit price, seller charges, and length of the credit period as decision variables (Abad & Jaggi, 2003). Chung & Huang (2003) developed the Goyal's (Goyal, 1985) model for Economical Production Quantity (EPQ) with delayed payments. Then, Chung & Huang (2006) extended a model to consider the defective items in the EPQ model with delayed payments and assumed the shortage is not allowed and the demand rate is constant. Chung (2009) studied an EOQ with deteriorating items and delays in payments. He assumed that the annual demand rate is constant, Shortages are not allowed, and the time horizon is infinite (Chung, 2009). Hu & Liu (2010) investigated the EOQ model with delays in payments and allowed shortages. They assumed that the unit selling price is not necessarily equal to the unit purchasing price and the demand is constant (Hu & Liu, 2010). Khanra et al. (2011) proposed an EOQ model with a constant rate of deterioration and time dependent demand and delay payments. Then, Min et al. (2012) examined the EPQ model with deteriorating products and delayed payments, and demand dependent on the retailer's stock level. Li et al. (2014) studied the joint order of several retailers who buy similar goods from one supplier. Delays in payments were allowed, and the results showed that forming a large coalition of retailers was socially beneficial (Li et al., 2014). Sadeghi et al. (2016) considered an inventory control model with discrete demand, stochastic lead time, and periodic order quantity (POQ) policy. They assumed the shortage was permitted and that a fixed percentage of items would defect during production. Patoghi & Setak (2018) considered an EOQ model for noninstantaneous deteriorating items without shortage. They assumed that the demand depends on the frequency of advertisement and the selling price. Chaudhari et al. (2020) considered a single product with seasonal demand and time-dependent deteriorating items. They assumed that the retailer could pay the purchase cost before delivery (Chaudhari et al., 2020). Supakar & Mahato (2020) developed a deteriorating EPQ model for a single item with delayed payment. However, they assumed the shortage was not allowed. Sadeghi et al. (2021) proposed an optimal integrated production-inventory model with multi-delivery. An EPQ Model for Deteriorating Products with Delayed Payments and Shortage ordered (Sadeghi et al., 2021). Duary et al. (2021) assumed that the suppliers used an offer in the price discounts for payments made by their retailers. They assumed the backlogged shortage was allowed. Sundararajan et al. (2021) analyzed partially backlogged shortages in the EOQ inventory model. However, there are no papers considering delayed payments for the EPQ system of finite production, deteriorating items with shortages. This study tries to fill this gap by considering variable demands.

Assumptions and notations:

The following assumptions and notations have been used in this full paper.

Assumptions

1. A single item is produced over a prescribed period of T unit of time.

2. The replenishment occurs at an finite rate.

3. The demand rate function D(t) is deterministic and is a known function of time and it is given by

$$D(t) = \begin{cases} ae^{-\lambda t} & I(t) > 0 \\ a & I(t) \le 0 \end{cases}$$

where $a > 0$ & $\lambda > 0$.

4. A linear deterioration cost function $P(t) = \varphi(t - t_d),\ \ t \ge t_d$, which gives the cost of keeping one unit of product in stock until age t, where t_d be the time period at which deterioration of product starts and φ is constant**.**

5. Shortages are allowed and are subject to partial backlogging, the backlogged rate is defined to be $\frac{1}{1+\ \delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant and $t_1 \le t < T$. The backlogging rate varies inversely as the waiting time for the next replenishment.

6. Lead time is zero.

7. When $t \ge M$, the payment by the retailer is settled at $t = M$. Beyond the fixed credit period, the retailer begins paying the interest charges on the items in stock at rate Ip. Before the settlement of the replenishment amount, the retailer can use the sales revenue to earn the interest at annual rate Ie , where $Ip \ge Ie$. When $t \le M$, the account is settled at $t = M$ and the retailer does not pay any interest charge. Alternatively, the retailer can accumulate revenue and earn interest until the end of the trade credit period.

NOTATIONS

The following notations are used in this full paper

p	production cost per unit item
p_1	selling price per unit item
I_p	The interest charged per dollar in stocks per year

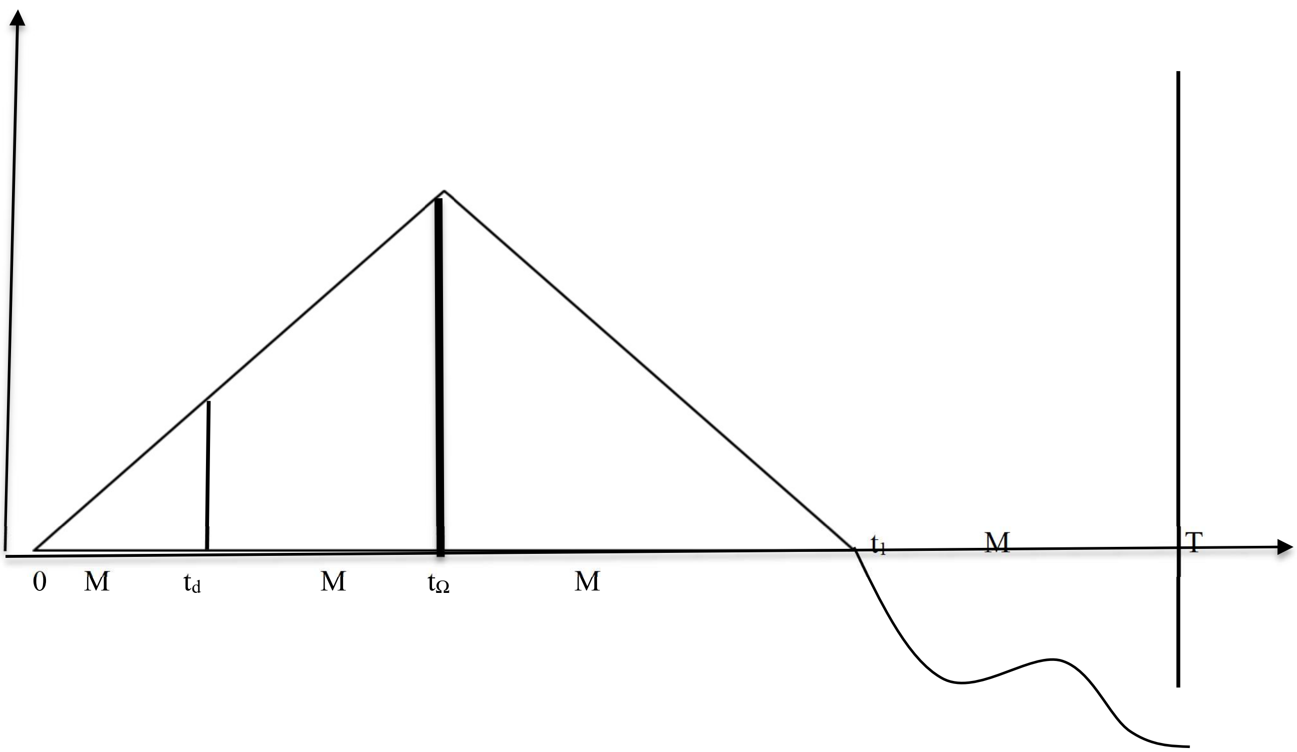
I_e	The interest earned per dollar per year
M	Permissible delay in paying the purchased amount
P	Production rate (finite)
Q	Number of items produced per production run
D(t)	Demand rate
h	Inventory holding cost per unit per unit time
K	Setup cost
t_d	Time period at which deterioration of product start
t_Ω	Time period at which production stopped
q	Maximum inventory level at time t_Ω
T	Length of replenishment cycle, which will not exceed product lifetime
s	Shortage cost per unit time
φ	deterioration cost
θ	Rate of deterioration
π	Opportunity cost
δ	Backlogging parameter
t_1	The optimum time at which the inventory reaches zero and inventory starts to accumulate
TC(t)	Total cost per unit time

MODEL FORMULATION

EOQ Model for finite production with shortages

The production starts at time t = 0 and the items produced always greater than the demand. Initially the stock is zero, the production (P) starts with a finite rate and the demand rate is D. If "Q" be the number of items produced per production run and the production continue for a period t_Ω. The production and the supply starts simultaneously and the inventory increases with a production rate minus the demand rate per unit per time till the stock reaches its maximum at time t_Ω. At this point the production is terminated and the stock on hand decreases due to combined effects of demand and deterioration in the interval $[t_\Omega, t_1]$. At $t_1$the inventory level reaches zero.

We consider an economical production system with a fixed production rate in which the demand depends on the time. The product is deteriorating, and the rate of deterioration is a fixed percentage of the inventory level. The manufacturer sells its products to the retailer with a trade credit option. Delay payment times can be at the times, where the time is before or after deterioration the product in stock or after the maximum inventory level or at the time of shortage. Accordingly, there are four possible payment intervals. Fig.(1) show the trend of inventory level behaviour over time. Furthermore, the shortages are allowed and are partially backlogged.



Variations of Inventory are given by the following equations

$$\frac{dI_1(t)}{dt} = P - ae^{-\lambda t}, \qquad 0 \le t \le t_d$$

$$\frac{dI_2(t)}{dt} + \theta\, I_2(t) = P - ae^{-\lambda} \qquad t_d < t \le t_\Omega$$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -ae^{-\lambda t} \qquad t_\Omega < t \le t_1$$

$$\frac{dI_4(t)}{dt} = \frac{-a}{1+\delta(T-t)} \qquad t_1 < t \le T$$

Using boundary conditions

$I_1(0) = 0$, $I_2(t_\Omega) = q$, $I_3(t_1) = 0$, $I_4(T) = 0$

$$I_1(t) = \frac{ae^{-\lambda t}}{\lambda} + Pt - \frac{a}{\lambda}$$

$$I_2(t) = \frac{P}{\theta} - \frac{ae^{-\theta t - t(\lambda-\theta)}}{-\lambda+\theta} - \frac{e^{(-\theta\ )}(P\lambda - \theta P + ae^{-\lambda t_\Omega}\theta - q\theta\lambda + q\theta^2)}{e^{-\theta t_d t_\Omega}\theta(\lambda-\theta)}$$

$$I_3(t) = e^{-\theta t}\left[\frac{ae^{-t(\lambda-\theta)}}{\lambda-\theta} - \frac{ae^{-t_1(\lambda-\theta)}}{\lambda-\theta}\right]$$

$$I_4(t) = \frac{a(log(1+\delta(T-t)) - log(1+\delta(T-t_1)))}{\delta}$$

From the continuity of $I_1(t_d) = I_2(t_d)$ gives the inventory level at t_Ω

$$q = -\frac{ae^{-\lambda t_d}\theta^2 e^{-\theta t_\Omega} - Pt_d\theta\lambda e^{-\theta t_\Omega}(\lambda-\theta) + ae^{-\theta t_\Omega}\theta\lambda(\lambda-\theta) + P\lambda e^{-\theta t_\Omega}(\lambda-\theta) - P\lambda e^{-\theta t_d}(\lambda-\theta) - \lambda e^{-\theta t_d} ae^{-\lambda t_\Omega}\theta}{e^{-\theta t_d}\theta\lambda(\lambda-\theta)}$$

Production cost

The production cost per unit item is given by

PC = pQ

Setup cost

The setup cost of inventory for the period is given by

S = K

Holding cost

The holding cost per cycle is given by

$$H = h\int_0^{t_d} I_1(t)dt + h\int_{t_d}^{t_\Omega} I_2(t)dt + h\int_{t_\Omega}^{t_1} I_3(t)dt$$

$$\mathbf{H} = \frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)}(\boldsymbol{h}(-2a\,\theta\lambda(e^{\lambda t_1+\theta t_\Omega} - e^{\theta t_1+\lambda t_\Omega}\lambda - e^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)e^{-(t_\Omega-t_1)\lambda-\theta t_\Omega} - 2e^{t_\Omega(-\lambda+\theta)}a\,\lambda e^{(t_\Omega-t_d)\lambda-\theta t_\Omega}\theta^2 - 2\left(((t_d-t_\Omega)P+q)\theta - P\right)\lambda^2(-\lambda+\theta) - 2e^{t_\Omega(-\lambda+\theta)}\lambda^2(-\lambda+\theta)(-q\,\theta + P)e^{-\theta t_d+\lambda t_\Omega} + \theta(2\lambda a\left((-\lambda+\theta)e^{-\theta t_\Omega} + \lambda e^{-\theta t_d}\right)e^{t_\Omega(-\lambda+\theta)} + \theta(-\lambda+\theta)(2a - 2ae^{-\lambda t_d} + Pt_d^{\,2}\lambda^2 - 2at_d\,\lambda))))$$

Shortage cost

Now the cost of shortage for the period (t_1,T) is given by

$$SC = s\int_{t_1}^{T} -I_4(t)dt$$

$$= -sa\left[\frac{\left(log(1+\delta(T-t_1)) - \delta(T-t_1)\right)}{\delta^2}\right]$$

Opportunity cost

The opportunity cost due to sales lost during the replenishment cycle for the period (t_1,T) is given by OC = $\pi\int_{t_1}^{T}[a - \frac{a}{1+\delta(T-t)}]\,dt$

$$= \pi a\{(T-t_1) + \frac{log(1+\delta(T-t_1))}{\delta}\}$$

Deterioration cost

A linear deterioration cost function $P(t) = \varphi(t-t_d),\ t \geq t_d$, which gives the cost of keeping one unit of product in stock until age t, where t_d be the time period at which deterioration of product starts and φ is constant. If linear deterioration cost function is used, then the cost due to deterioration of products during the period (t_d, t_1) is given by

$$DC = p\{\int_{t_d}^{t_\Omega}[\varphi(t-t_d)\theta I_2(t)dt + \int_{t_\Omega}^{t_1}[\varphi(t-t_d)\theta I_3(t)dt\}$$

$$DC = \frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)}\Big(\boldsymbol{p}\Big((\lambda^2(-\lambda+\theta)(((t_d-t_\Omega)\boldsymbol{P}+\boldsymbol{2q})(t_d-t_\Omega)\boldsymbol{\theta^2}+\big((-\boldsymbol{2}t_d+\boldsymbol{2}t_\Omega)\boldsymbol{P}-\boldsymbol{2q}\big)\boldsymbol{\theta}+\boldsymbol{2P}\Big)\boldsymbol{e}^{(t_d+t_\Omega)\lambda+\theta t_d}-\boldsymbol{2}\lambda^2(-\lambda+\theta)(-q\theta+P)\boldsymbol{e}^{(t_d+t_\Omega)\lambda+\theta t_\Omega}-\boldsymbol{2\theta a}\Big(\boldsymbol{\theta^2}e^{\theta t_d+\lambda t_\Omega}-\boldsymbol{\lambda^2}e^{\theta t_\Omega+\lambda t_d}+(-\lambda+\theta)e^{t_d(\lambda+\theta)}\big((-1+(t_d-t_\Omega)\lambda)\theta-\lambda\big)\Big)\Big)e^{(-t_d-t_\Omega)\lambda-\theta t_d}+2e^{(-t_1-t_\Omega)\lambda-\theta t_\Omega}\theta a\Big(\Big(-1+(t_d-t_\Omega)\boldsymbol{e}^{\boldsymbol{\theta}t_1+\lambda t_\Omega}-\boldsymbol{\lambda^2}(-\boldsymbol{1}+(t_d-t_\Omega)\boldsymbol{\theta})\boldsymbol{e}^{\boldsymbol{\theta}t_1+\lambda t_\Omega}-\boldsymbol{e}^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)\big((-1+(t_d-t_1)\lambda)\theta-\lambda\big)\Big)\Big)$$

Now, we have to consider M which is the permissible delay in settling the accounts offered by the manufacturer. There are four possibilities.

CASE 1: $0 < M \leq t_d$

$IP_1 = p\, I_p \left[\int_M^{t_d} I_1(t)dt + \int_{t_d}^{t_\Omega} I_2(t)dt + \int_{t_\Omega}^{t_1} I_3(t)dt \right]$

$$IP1 = -\frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)}\Big(\Big(2a\lambda\theta\left(c^{t_1\lambda+\theta t_\Omega}\theta - c^{\theta t_1+\lambda t_\Omega}\lambda - c^{t_\Omega(\lambda+\theta)}(-\lambda+\theta)\right)c^{(-t_\Omega-t_1)\lambda-\theta t_\Omega} + 2c^{t_\Omega(-\lambda+\theta)}\lambda a c^{(t_\Omega-t_d)\lambda-\theta}\theta^2 + 2c^{t_\Omega(-\lambda+\theta)}\lambda^2\left(((t_d-t_\Omega)P+q)\theta-P\right)(-\lambda+\theta)c^{-t_\Omega(-\lambda+\theta)} + 2c^{t_\Omega(-\lambda+\theta)}\lambda^2(-\lambda+\theta)(-q\theta+P)c^{-\theta t_d+\lambda t_\Omega} + \theta\Big(-2a\lambda\left((-\lambda+\theta)c^{-\theta t_\Omega}+\lambda c^{-\theta t_d}\right)c^{t_\Omega(-\lambda+\theta)} + \theta(-\lambda+\theta)\big(-2ac^{-\lambda M}+2ac^{-\lambda t_d}+\lambda(P(M+t_d)\lambda-2a)(M-t_d)\big)\Big)\Big)p\, Ip\Big)$$

$IE_1 = p_1\, I_e \int_0^M ae^{-\lambda t}t\,dt$

$IE_1 = -\frac{p_1 I_e\, a(-1+e^{-\lambda M}+e^{-\lambda M}\lambda M)}{\lambda^2}$

The total cost per unit time is

$TC_1(t) = \frac{PC+S+H+DC+SC+OC-IE1+IP1}{T}$

$$= \frac{1}{T}\Big[pQ+K+\frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)}\Big(\boldsymbol{h}(-\boldsymbol{2}a\,\theta\lambda(e^{\lambda t_1+\theta t_\Omega}-e^{\theta t_1+\lambda t_\Omega}\lambda-e^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)e^{-(t_\Omega-t_1)\lambda-\theta t_\Omega}-2e^{t_\Omega(-\lambda+\theta)}a\lambda e^{(t_\Omega-t_d)\lambda-\theta t_\Omega}\theta^2-2\left(((t_d-t_\Omega)P+q)\theta-P\right)\lambda^2(-\lambda+\theta)-2e^{t_\Omega(-\lambda+\theta)}\lambda^2(-\lambda+\theta)(-q\theta+P)e^{-\theta t_d+\lambda t_\Omega}+\theta(2\lambda a\left((-\lambda+\theta)e^{-\theta t_\Omega}+\lambda e^{-\theta t_d}\right)e^{t_\Omega(-\lambda+\theta)}+\theta(-\lambda+\theta)(2a-2ae^{-\lambda t_d}+Pt_d{}^2\lambda^2-2at_d\lambda))))+\frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)}\Big(\boldsymbol{p}\Big((\lambda^2(-\lambda+\theta)(((t_d-t_\Omega)\boldsymbol{P}+\boldsymbol{2q})(t_d-t_\Omega)\boldsymbol{\theta^2}+\big((-\boldsymbol{2}t_d+\boldsymbol{2}t_\Omega)\boldsymbol{P}-\boldsymbol{2q}\big)\boldsymbol{\theta}+\boldsymbol{2P}\Big)\boldsymbol{e}^{(t_d+t_\Omega)\lambda+\theta t_d}-\boldsymbol{2}\lambda^2(-\lambda+\theta)(-q\theta+P)\boldsymbol{e}^{(t_d+t_\Omega)\lambda+\theta t_\Omega}-\boldsymbol{2\theta a}\Big(\boldsymbol{\theta^2}e^{\theta t_d+\lambda t_\Omega}-\boldsymbol{\lambda^2}e^{\theta t_\Omega+\lambda t_d}+(-\lambda+\theta)e^{t_d(\lambda+\theta)}\big((-1+(t_d-t_\Omega)\lambda)\theta-\lambda\big)\Big)\Big)e^{(-t_d-t_\Omega)\lambda-\theta t_d}+2e^{(-t_1-t_\Omega)\lambda-\theta t_\Omega}\theta a\Big(\Big(-1+(t_d-t_\Omega)\boldsymbol{e}^{\boldsymbol{\theta}t_1+\lambda t_\Omega}-\boldsymbol{\lambda^2}(-\boldsymbol{1}+(t_d-t_\Omega)\boldsymbol{\theta})\boldsymbol{e}^{\boldsymbol{\theta}t_1+\lambda t_\Omega}-\boldsymbol{e}^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)\big((-1+(t_d-t_1)\lambda)\theta-\lambda\big)\Big)\Big)$$

-sa$\frac{(log(1+\delta(T-t_1))-\delta(T-t_1))}{\delta^2}$]+ $\pi a\left\{(T-t_1)+\frac{log(1+\delta(T-t_1))}{\delta}\right\}-[-\frac{p_1 I_e\, a(-1+e^{-\lambda M}+e^{-\lambda M}\lambda M)}{\lambda^2}$]

+[$-\frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)\delta^2}$ $(I_p p(2\lambda\left(e^{t_1\lambda+\theta t_\Omega}\theta-e^{t_\Omega t_1+\theta y}\lambda-e^{t_\Omega(\lambda+\theta)}(-\lambda+\theta)\right)\theta a\delta^2 e^{(-t_\Omega-t_1)\lambda-\theta t_\Omega}+$ $2e^{t_\Omega(-\lambda+\theta)}\lambda\delta^2 a e^{(t_\Omega-t_d)\lambda-\theta t_\Omega}\theta^2-2a\lambda^2\theta^2(-\lambda+\theta)\log(1+\delta(T-t_1))+\delta(2\lambda^2\delta(-\lambda+\theta)\Big(((t_d-$ $t_\Omega)P+q)\theta-P\Big)+2\delta e^{t_\Omega(-\lambda+\theta)}\lambda^2(-\lambda+\theta)(P-q\theta)e^{t_\Omega\lambda-\theta t_d}$ + $\theta(-2\lambda a\,\delta\Big((-\lambda+\theta)e^{-\theta t_\Omega}+$ $e^{-\theta t_d}\lambda\Big)e^{t_\Omega(-\lambda+\theta)}+\theta(-2a\,\delta e^{-\lambda}+2a\,\delta e^{-\lambda t_d}+\lambda(((-t_d{}^2+M^2)P\delta+2a(T-t_1)-2\delta a(M-$ $t_d))(-\lambda+\theta)))))]$ ....(1)

CASE 2: $t_d < M \le t_\Omega$

$IP_2 = p\, I_p\, [\int_M^{t_\Omega} I_2(t)dt + \int_{t_\Omega}^{t_1} I_3(t)dt]$

$$IP2=-\frac{1}{\lambda(-\lambda+\theta)\theta^2}\Big(p\,Ip\Big(-\theta\big(c^{t_1\lambda+\theta t_\Omega}\theta-c^{\theta t_1+\lambda t_\Omega}\lambda-c^{t_\Omega(\lambda+\theta)}(-\lambda+\theta)\big)ac^{(-t_\Omega-t_1)\lambda-\theta t_\Omega}+c^{\theta t_\Omega}\Big(ac^{-M\lambda-\theta t_\Omega}\theta^2-ac^{-t_1\lambda-\theta t_\Omega}\theta^2+\Big(-c^{-\theta M-\lambda t_\Omega}a\theta+c^{-\theta t_1-\lambda t_\Omega}a\theta+\Big((-q\theta+P)c^{-\theta M}+(q\theta-P)c^{-\theta t_1}+P\theta c^{-\theta t_\Omega}(M-t_1)\Big)(-\lambda+\theta)\Big)\lambda\Big)\Big)\Big)$$

$IE_2 = p_1\, I_e\int_0^M ae^{-\lambda t}t\,dt$

$= -\frac{p_1 I_e\, a(-1+e^{-\lambda M}+e^{-\lambda M}\lambda M)}{\lambda^2}$

The total cost per unit time is

$TC_2(t) = \frac{PC+S+H+DC+SC+OC-IE2+IP2}{T}$

$\frac{1}{T}$ [pQ + K + $\frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)}(\boldsymbol{h}(-2a\,\theta\lambda(e^{\lambda t_1+\theta t_\Omega}-e^{\theta t_1+\lambda t_\Omega}\lambda-e^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)e^{-(t_\Omega-t_1)\lambda-\theta t_\Omega}-$ $2e^{t_\Omega(-\lambda+\theta)}a\,\lambda e^{(t_\Omega-t_d)\lambda-\theta t_\Omega}\theta^2-2\Big(((t_d-t_\Omega)P+q)\theta-P\Big)\lambda^2(-\lambda+\theta)-2e^{t_\Omega(-\lambda+\theta)}\lambda^2(-\lambda+\theta)(-q\,\theta+$ $P)e^{-\theta t_d+\lambda t_\Omega}+\theta(2\lambda a\Big((-\lambda+\theta)e^{-\theta t_\Omega}+\lambda e^{-\theta t_d}\Big)e^{t_\Omega(-\lambda+\theta)}+\theta(-\lambda+\theta)(2a-2ae^{-\lambda t_d}+Pt_d{}^2\lambda^2$ $-$ $2at_d\ \lambda))))+$ $=\frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)}\Big(\boldsymbol{p}\Big((\lambda^2(-\lambda+\theta)(((t_d-t_\Omega)\boldsymbol{P}+\boldsymbol{2q})(t_d-t_\Omega)\boldsymbol{\theta^2}+((-\boldsymbol{2}t_d+\boldsymbol{2}t_\Omega)\boldsymbol{P}-\boldsymbol{2q})\boldsymbol{\theta}+$ $\boldsymbol{2P}\Big)\boldsymbol{e}^{(t_d+t_\Omega)\lambda+\theta t_d}-\boldsymbol{2}\lambda^2(-\lambda+\theta)(-q\theta+P)\boldsymbol{e}^{(t_d+t_\Omega)\lambda+\theta t_\Omega}-\boldsymbol{2\theta a}\Big(\boldsymbol{\theta^2}e^{\theta t_d+\lambda t_\Omega}-\lambda^2 e^{\theta t_\Omega+\lambda t_d}+(-\lambda+$ $\theta)e^{t_d(\lambda+\theta)}\,((-1+(t_d-t_\Omega)\lambda)\theta-\lambda)\Big)\Big)e^{(-t_d-t_\Omega)\lambda-\theta t_d}+2e^{(-t_1-t_\Omega)\lambda-\theta t_\Omega}\theta a\Big(\Big(-1+(t_d-t_\Omega)\boldsymbol{e}^{\boldsymbol{\theta}t_1+\lambda t_\Omega}-$ $\boldsymbol{\lambda^2}(-\boldsymbol{1}+(t_d-t_\Omega)\boldsymbol{\theta})\boldsymbol{e}^{\boldsymbol{\theta}t_1+\lambda t_\Omega}-\boldsymbol{e}^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)\big((-1+(t_d-t_1)\lambda)\theta-\lambda\big)\Big)\Big)$

-sa$\frac{(log(1+\delta(T-t_1))-\delta(T-t_1))}{\delta^2}$]+ $\pi a\left\{(T-t_1)+\frac{log(1+\delta(T-t_1))}{\delta}\right\}-$ [- $\frac{p_1 I_e\, a(-1+e^{-\lambda M}+e^{-\lambda M}\lambda M)}{\lambda^2}$]+ $\frac{pI_p}{\lambda\theta^2(-\lambda+\theta)\delta^2}((-\delta^2\theta(e^{\lambda t_1+\theta t_\Omega}\theta-\lambda e^{\lambda t_\Omega+\theta t_1}-e^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)ae^{\lambda(-t_1-t_\Omega)-\theta t_\Omega}-\theta^2(-\lambda+\theta)\,(1+(T-$ $t_1)\delta)\lambda a log(1+(T-t_\Omega)$ $\delta+\delta(a\theta^2\lambda(-\lambda+\theta)(T-t_\Omega)\log(1+\delta(T-t_1))+\delta e^{\theta t_1}a(e^{-M\lambda-\theta t_\Omega}-$ $e^{-t_1\lambda-\theta t_\Omega})\theta^2+\lambda\Big(\delta e^{\theta t_\Omega}(-e^{-t_\Omega\lambda-\theta M}+e^{-t_\Omega\lambda-\theta t_1})a\theta+(-\lambda+\theta)\Big(\delta e^{\theta t_\Omega}(-q\theta+P)(e^{-\theta M}-e^{-\theta t_1})+$

$\theta\left(\delta e^{\theta t_\Omega}P(M-t_1)e^{-\theta t_\Omega}+a\theta(\mathrm{T}-t_\Omega)))\right)\Big)\Big)]\ldots\ldots$*(2)*

CASE 3: $t_\Omega < \mathrm{M} \leq t_1$

$IP_3 = p\, I_p \int_M^{t_1} I_3(t)dt$

$$IP3 = \frac{\left(c^{(t_1+M)\lambda+\theta(M+t_\Omega)}\theta - c^{(t_\Omega+t_1)\lambda+\theta(M+t_\Omega)}\theta + \lambda\left(c^{(M+t_\Omega)\lambda+\theta(t_\Omega+t_1)} - c^{(M+t_\Omega)\lambda+(t_1+M)\theta}\right)\right)ac^{(-M-t_1-t_\Omega)\lambda-\theta(M+t_\Omega)}p\, Ip}{(-\lambda+\theta)\lambda\theta}$$

$IE_3 = p_1\, I_e \int_0^M ae^{-\lambda t}tdt$

$= -\frac{p_1 I_e\, a(-1+e^{-\lambda M}+e^{-\lambda M}\lambda M)}{\lambda^2}$

The total cost per unit time is

$TC_3(t) = \frac{PC+S+H+DC+SC+OC-IE3+IP3}{T}$

$\frac{1}{T}\,[pQ + K + \frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{1}}{\lambda^2\theta^2(-\lambda+\theta)}(\boldsymbol{h}(-\mathbf{2}a\,\theta\lambda(e^{\lambda t_1+\theta t_\Omega} - e^{\theta t_1+\lambda t_\Omega}\lambda - e^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)e^{-(t_\Omega-t_1)\lambda-\theta t_\Omega} - 2e^{t_\Omega(-\lambda+\theta)}a\,\lambda e^{(t_\Omega-t_d)\lambda-\theta t_\Omega}\theta^2 - 2\left(((t_d-t_\Omega)P+q)\theta - \mathrm{P}\right)\lambda^2(-\lambda+\theta) - 2e^{t_\Omega(-\lambda+\theta)}\lambda^2(-\lambda+\theta)(-q\,\theta + \mathrm{P})e^{-\theta t_d+\lambda t_\Omega} + \theta(2\,\lambda a\left((-\lambda+\theta)e^{-\theta t_\Omega}+\lambda e^{-\theta t_d}\right)e^{t_\Omega(-\lambda+\theta)} + \theta(-\lambda+\theta)(2a - 2ae^{-\lambda t_d} + Pt_d{}^2\lambda^2 - 2at_d\,\lambda))))+ = \frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{1}}{\lambda^2\theta^2(-\lambda+\theta)}\Big(\boldsymbol{p}\left((\lambda^2(-\lambda+\theta)(((t_d-t_\Omega)\boldsymbol{P}+\mathbf{2}\boldsymbol{q})(t_d-t_\Omega)\boldsymbol{\theta}^{\mathbf{2}} + ((-\mathbf{2}t_d+\mathbf{2}t_\Omega)\boldsymbol{P}-\mathbf{2}\boldsymbol{q})\boldsymbol{\theta} + \mathbf{2}\boldsymbol{P}\right)\boldsymbol{e}^{(t_d+t_\Omega)\lambda+\theta t_d} - \mathbf{2}\lambda^2(-\lambda+\theta)(-q\theta+P)\boldsymbol{e}^{(t_d+t_\Omega)\lambda+\theta t_\Omega} - \mathbf{2}\boldsymbol{\theta a}\left(\boldsymbol{\theta}^{\mathbf{2}}e^{\theta t_d+\lambda t_\Omega} - \lambda^2 e^{\theta t_\Omega+\lambda t_d} + (-\lambda+\theta)e^{t_d(\lambda+\theta)}\left((-1+(t_d-t_\Omega)\lambda)\theta-\lambda\right)\right)\Big)e^{(-t_d-t_\Omega)\lambda-\theta t_d} + 2e^{(-t_1-t_\Omega)\lambda-\theta t_\Omega}\theta a(\left(-1+(t_d-t_\Omega)\boldsymbol{e}^{\theta t_1+\lambda t_\Omega} - \lambda^{\mathbf{2}}(-\mathbf{1}+(t_d-t_\Omega)\boldsymbol{\theta})\boldsymbol{e}^{\boldsymbol{\theta}t_1+\lambda t_\Omega} - \boldsymbol{e}^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)\left((-1+(t_d-t_1)\lambda)\theta-\lambda\right)\right))$

$-sa\frac{\left(log(1+\delta(T-t_1))-\delta(T-t_1)\right)}{\delta^2}] + \pi a\left\{(T-t_1)+\frac{log(1+\delta(T-t_1))}{\delta}\right\} + \pi a\left\{(T-t_1)+\frac{log(1+\delta(T-t_1))}{\delta}\right\} - \left[-\frac{p_1 I_e\, a(-1+e^{-\lambda M}+e^{-\lambda M}\lambda M)}{\lambda^2}\right] + \frac{\mathbf{1}}{\lambda\theta(-\lambda+\theta)\delta^2}[\boldsymbol{ap}(\left(\boldsymbol{e}^{(t_1+\boldsymbol{M})\lambda+\theta(M+t_\Omega)}\theta - \boldsymbol{e}^{(t_1+t_\Omega)\lambda+\theta(M+t_\Omega)}\theta + \lambda\left(\boldsymbol{e}^{(t_\Omega+\boldsymbol{M})\lambda+\theta(y+t_\Omega)} - \boldsymbol{e}^{(t_\Omega+\boldsymbol{M})\lambda+\theta(M+t_1)}\right)\right)\delta^2\boldsymbol{e}^{-(t_\Omega+t_1+\boldsymbol{M})\lambda-\theta(M+t_\Omega)} + (-\lambda+\theta)\theta\left((1+(\mathrm{T}-t_\Omega)\delta)\log(1+(T-t_\Omega)\delta) - \delta(1+\log(1+\delta(T-t_1)))(T-t_\Omega)\lambda\right)\boldsymbol{I_p})$ ***……(3)***

CASE 4: $t_1 < \mathrm{M} \leq \mathrm{T}$

$IE_4 = p_1\, I_e\left[\int_o^{t_1} ae^{-\lambda t}tdt + (M-t_1)\int_0^M ae^{-\lambda t}dt\right]$

$= -\frac{\left(((\mathrm{Ie}\,p_1\,t_1+M-t_1)\lambda+\mathrm{Ie}\,p_1)e^{-\lambda t_1}+(t_1-M)\lambda-\mathrm{Ie}\,p_1\right)a}{\lambda^2}$

The total cost per unit time is

$TC_4(t) = \frac{PC+S+H+DC+SC+OC-IE}{T}$

$\frac{1}{T}\,[pQ + K + \frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{1}}{\lambda^2\theta^2(-\lambda+\theta)}(\boldsymbol{h}(-\mathbf{2}a\,\theta\lambda(e^{\lambda t_1+\theta t_\Omega} - e^{\theta t_1+\lambda t_\Omega}\lambda - e^{(\lambda+\theta)t_\Omega}(-\lambda+\theta)e^{-(t_\Omega-t_1)\lambda-\theta t_\Omega} - 2e^{t_\Omega(-\lambda+\theta)}a\,\lambda e^{(t_\Omega-t_d)\lambda-\theta t_\Omega}\theta^2 - 2\left(((t_d-t_\Omega)P+q)\theta - \mathrm{P}\right)\lambda^2(-\lambda+\theta) - 2e^{t_\Omega(-\lambda+\theta)}\lambda^2(-\lambda+\theta)(-q\,\theta +$

$P)e^{-\theta t_d+\lambda t_\Omega}+\theta(2\lambda a\left((-\lambda+\theta)e^{-\theta t_\Omega}+\lambda e^{-\theta t_d}\right)e^{t_\Omega(-\lambda+\theta)}+\theta(-\lambda+\theta)(2a-2ae^{-\lambda t_d}+Pt_d{}^2\lambda^2-2at_d\lambda))))$

$+\frac{1}{2}\frac{1}{\lambda^2\theta^2(-\lambda+\theta)}((((\left((t_d-t_\Omega)\left((t_d-t_\Omega)\boldsymbol{P}+\boldsymbol{2q}\right)\theta^2+\left((-2t_d+2t_\Omega)P-2q\right)\theta+2P\right)(-\lambda+\theta)\lambda^2e^{(t_\Omega+t_d)\lambda+\theta t_d}-2\lambda^2(-\lambda+\theta)(-q\theta+P)e^{(t_\Omega+t_d)\lambda+\theta t_\Omega}-2\theta a\left(\theta^2e^{t_\Omega\lambda+\theta t_d}-\lambda^2e^{\lambda t_d+\theta t_\Omega}+e^{t_d(\lambda+\theta)}(-\lambda+\theta)\left((-1+(t_d-t_\Omega)\theta-\lambda)\right)\right)e^{(-t_\Omega-t_d)\lambda-\theta t_d}+2(\theta^2(-1+(t_d-t_\Omega)\lambda e^{\lambda t_1+\theta t_\Omega}-\lambda^2(-1+(t_d-t_\Omega)\theta)e^{\lambda t_\Omega+\theta t_1}-(-\lambda+\theta)\left((-1+(t_d-t_1)\lambda)\theta-\lambda\right)e^{t_\Omega(\lambda+\theta)}\theta ae^{(-t_\Omega-t_1)\lambda-\theta t_\Omega})\varphi)$-

$sa\frac{\left(log(1+\delta(T-t_1))-\delta(T-t_1)\right)}{\delta^2}]+\pi a\left\{(T-t_1)+\frac{log(1+\delta(T-t_1))}{\delta}\right\}$

$-sa\frac{\left(log(1+\delta(T-t_1))-\delta(T-t_1)\right)}{\delta^2}]+\pi a\left\{(T-t_1)+\frac{log(1+\delta(T-t_1))}{\delta}\right\}-$

$(-\frac{\left(((\text{Ie}\,p_1\,t_1+M-t_1)\lambda+\text{Ie}\,p_1)e^{-\lambda t_1}+(t_1-M)\lambda-\text{Ie}\,p_1\right)a}{\lambda^2})$ ………*(4)*

For the cases (1) to (4) the total cost is found from the following algorithm.

The necessary conditions for the total annual cost $TC_i(t_1,T)$ is minimum with respect to t_1 and T are $\frac{\partial TC_i(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC_i(t_1,T)}{\partial T}=0$ …(5)

Such that they have to satisfy the following conditions

$\frac{\partial TC_i(t_1,T)}{\partial t_1^2}>0$, $\frac{\partial TC_i(t_1,T)}{\partial T^2}>0$ and

$\{(\frac{\partial TC_i(t_1,T)}{\partial t_1^2})(\frac{\partial TC_i(t_1,T)}{\partial T^2})-\frac{\partial^2 TC_i(t_1,T)}{\partial t_1\partial T}\}>0$ at $t_1=t_1^*$ and $T=T^*$ …(6)

Algorithm [for cases (1 to 4)]

Step 1: Start

Step 2: Evaluate $\frac{\partial TC_i(t_1,T)}{\partial t_1}$ and $\frac{\partial TC_i(t_1,T)}{\partial T}$

Step 3: Solve the simultaneous equations $\frac{\partial TC_i(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC_i(t_1,T)}{\partial T}=0$ and find the values of t_1 *and* T by fixing t_d *and* t_Ω and initializing the values of P,k,h,a,q,c, $\varphi,\theta,\delta,\pi,p,p_1,\lambda$

Step 4: Choosing one set of solution from 3.

Step 5: If the values in equation (3) are greater than zero, then this set of solution is optimal and go to step 6. Otherwise go to step 4 with next set of solution obtained from step 3.

Step 6: Evaluate $TC_i\,(t_1^*,T^*)$

Step 7: End

For i = 1,2,3,4 for cases 1 to 4 respectively.

Numerical examples:

In order to illustrate the above procedure, we consider the following numerical examples

Consider an inventory system with the following data K=200/order, a= 20, $t_d=0.041$, $t_\Omega=0.082$, P = 40, $\varphi=3.14$, $\theta=0.08$, h = 2.5, $\pi=5$, $\delta=0.56$, s = 8, $I_p=0.13$, $I_e=0.12$, $\lambda=0.03$, p = 10 and $p_1=18$ in appropriate units. In the following examples from 1 to 4 all the above values are kept fixed and change of values in M according to the above cases from 1 to 4.

Example 1

Taking M = 0.02 in this case $M<t_d$, For this case, applying algorithm when i = 1, the optimal solution, $t_1=$

1.8385, T = 3.3782 and the total cost TC_1 = 262.40in dollars and
q = 17.519 units.

Example 2

Taking M = 0.061 in this case M $> t_d$, For this case, applying algorithm when i = 2, we get the optimal solution, $t_1 = 1.284$, T = 2.739 and the total cost TC_2 = 245.842 in dollars and
q = 17.519 units.

Example 3

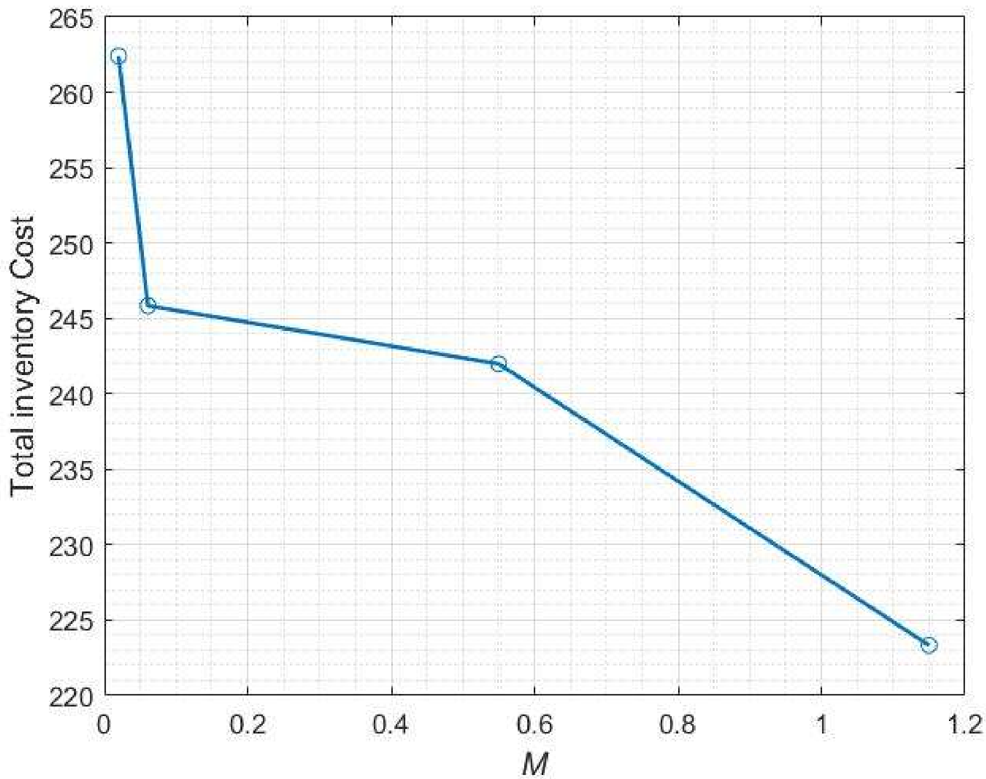
Taking M = 0.55, in this case M $> t_\Omega$, For this case, applying algorithm when i = 3, we get the optimal solution, $t_1 = 1.3053$, T =2.9502 and the total cost TC_3 = 241.99 in dollars and
q = 17.519 units.

Example 4

Taking M = 1.15, in this case M $> t_1$, here applying algorithm when i = 4, we get the optimal solution, $t_1 = 0.9538$, T = 3.7307 and the total cost TC_4 = 223.329 in dollars and
q = 17.519 units.

TABLE: Effect of M on Total inventory cost

Credit period M in days/year	**Total inventory cost in dollars**
0.02	**262.40**
0.061	**245.842**
0.55	**241.99**
1.15	**223.329**



Conclusion:

In this paper, a production inventory model for non-instantaneous deteriorating items with time dependent demand is developed. Production rate is postulated to be finite. In this paper the idea of continuous deterioration of utility for an individual product and a measure for the utility deterioration as a linear deterioration cost function with shortages are allowed and can be partially backlogging. Credit financing policy is allowed. The increasing value of M will result in a decrease of the total inventory cost. Numerical examples are given to illustrate the models. From the result, we could see that the increasing value of M, results in a significant decrease in the optimal inventory cost. That is to say the manufacturer permissible delay makes the retailer very lucrative that is the retailer is the most beneficiary if he gets longer permissible delay from the manufacturer.

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