Higher order of mixed positive and negative Extorial function using generalized mixed Difference operator

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Abstract

In this paper, we have found the higher order of mixed positive and negativeextorial function using generalized mixed difference operator, Suitable examplesare inserted to illustrate the main results.

Key words: Mixed difference operator, Positive extorial function, Negativeextorial function.

1. INTRODUCTION

The properties of difference equations are depends on the operator Δ defined as$\Delta f(k) = f(k+1) - f(k)$. The mixed difference operator Δ denoted by $\Delta_{q,l}$and itsinverse operator $\Delta_{q,l}^{-1}$. The factorial polynomial is used to define new mixed positiveand negative extorial functions. This extorial function obtain the higher order ofmixed positive and negative extorial function.

2. PRELIMINARIES

In this section, we focus on the basic definition of the $q,l-$ mixed extorial function.

Definition 2.1. The $q,l-$extorial function denoted as $e\left(k_{q,l}^{(n)}\right)$ is defined as,

$$e\left(k_{q,l}^{(n)}\right) = 1 + \frac{k_{q,l}^{(n)}}{1!} + \frac{k_{q,l}^{(2n)}}{2!} + \frac{k_{q,l}^{(3n)}}{3!} + \cdots \quad (1)$$

Definition 2.2. The $|l| \leq 1, |q| \leq 1,\ k \in \mathbb{N}$ the positive order $q,l-$extorial function denoted as $e_m(k_{q,l})$ is defined as,

$$e_m(k_{q,l}) = 1 + \frac{k_{q,l}^{(m)}}{m!} + \frac{k_{q,l}^{(2m)}}{2m!} + \frac{k_{q,l}^{(3m)}}{3m!} + \cdots \quad (2)$$

Definition 2.3. The negativeextorial function $|q| \leq 1$ and $|l| \leq 1$ is defined as,

$$e_{(-m)}\left(k_{q,l}\right) = 1 + \frac{1}{m!}\frac{1}{k_{q,l}^{(m)}} + \frac{1}{(2m)!}\frac{1}{k_{q,l}^{(2m)}} + \frac{1}{(3m)!}\frac{1}{k_{q,l}^{(3m)}} + \cdots \quad (3)$$

Definition 2.4. If q, l and n are three positive integers, then the generalized q, l polynomial factorial is defined as,

$$k_{q,l}^{(n)} = k\left(\frac{k}{q} - l\right)\left(\frac{k}{q^2} - 2l\right)\cdots\left(\frac{k}{q^{n-1}} - (n-1)l\right) \quad (4)$$

3. Higher Order of Mixed Positive Extorial Function

In this section, we define the positive order extorial function and establish the higher order values of m^{th}order extorial function.

Theorem 3.1. Let $|l| \le 1, |q| \le 1$ and $n, m$and $k \in \mathbb{N}$. Then,

$$\Delta_{q,l}^{n}\left(e_m\left(k_{q,l}\right)\right) = q^n l^n \sum_{t=1}^{\infty} \frac{k_{q,l}^{(tm-n)}}{(tm-n)!}, tm \neq n \quad (5)$$

Proof.

We have, $e_m\left(k_{q,l}\right) = 1 + \frac{k_{q,l}^{(m)}}{m!} + \frac{k_{q,l}^{(2m)}}{(2m)!} + \frac{k_{q,l}^{(3m)}}{(3m)!} + \cdots$

Multiply Δ on both sides,

$$\Delta\left(e_m\left(k_{q,l}\right)\right) = \Delta\left(1 + \frac{k_{q,l}^{(m)}}{m!} + \frac{k_{q,l}^{(2m)}}{(2m)!} + \frac{k_{q,l}^{(3m)}}{(3m)!} + \cdots\right).$$

$$\Delta_{q,l}\left(e_m\left(k_{q,l}\right)\right) = ql\left[\frac{k_{q,l}^{(m-1)}}{(m-1)!} + \frac{k_{q,l}^{(2m-1)}}{(2m-1)!} + \frac{k_{q,l}^{(3m-1)}}{(3m-1)!} + \cdots\right]$$

$$\Delta_{q,l}\left(e_m\left(k_{q,l}\right)\right) = ql \sum_{t=1}^{\infty} \frac{k_{q,l}^{(tm-1)}}{(tm-1)!}$$

Proceeding like this, we get

$$\Delta_{q,l}^{n}\left(e_m\left(k_{q,l}\right)\right) = q^n l^n \sum_{t=1}^{\infty} \frac{k_{q,l}^{(tm-n)}}{(tm-n)!}, tm \neq n$$

□

Corollary 3.2. For any positive integer $q$and l, then

$$\Delta_{q,l}^{n}\left(e\left(k_{q,l}\right)\right) = q^n l^n \sum_{t=1}^{\infty} \frac{k_{q,l}^{(t-n)}}{(t-n)!}, t \neq n\,.$$

*Proof.*put $m = 1$ is above thm(3.1) we get,

$$\Delta_{q,l}^{n}\left(e\left(k_{q,l}\right)\right)=q^{n}l^{n}\sum_{t=1}^{\infty}\frac{k_{q,l}^{(t-n)}}{(t-n)!},t\neq n\,.$$

□

Example 3.3. Taking $n = 2, m = 5, k = 6, l = 2, q = 1$ substituting these valuesin equation (5)
In L.H.S

$$\begin{aligned}\Delta_{q,l}^{n}\left(e_m\left(k_{q,l}\right)\right)&=e_m(kq^2+2l)^{(m)}-2e_m(kq+l)^{(m)}+e_m\left(k_{q,l}\right)^{(n)}\\&=e_5(10)-2e_5+e_5(6)\\&=\left[1+\frac{(10)_{1,2}^{(5)}}{5!}\right]-2\left[1+\frac{(8)_{1,2}^{(5)}}{5!}\right]+\left[1+\frac{(6)_{1,2}^{(5)}}{5!}\right]=32\end{aligned}$$

In R.H.S

$$q^{n}l^{n}\sum_{t=1}^{\infty}\frac{k_{q,l}^{(tm-n)}}{(tm-n)!}=1^{2}2^{2}\left[\frac{48}{6}\right]=32$$

Thus we have verified the equation (5)

4. Higher Order of Mixed Negative Extorial Function

In this section, we define the negative order extorial function and establish the higher order values of m^{th}order extorial function.

Theorem 4.1. If $|l|\leq 1, |q|\leq 1, k<0$ and $\frac{1}{k_{q,l}^{(m)}}=k_{q,l}^{(-m)}$ then

$$\boldsymbol{\Delta_{q,l}^{-n}\left(e_{(-m)}\left(k_{q,l}\right)\right)}$$

$$=\frac{\mathbf{1}}{\boldsymbol{q^{n}l^{n}}}\left[\frac{k_{q,l}^{(n)}}{n!}(-\mathbf{1})^{\boldsymbol{n}}\sum_{t=1}^{\infty}\frac{\mathbf{1}}{(\boldsymbol{tm})!\,(\boldsymbol{tm}-\mathbf{1})(\boldsymbol{tm}-\mathbf{2})\cdots(\boldsymbol{tm}-\boldsymbol{n})\left(\frac{\boldsymbol{k}}{\boldsymbol{q^{n}}}-\boldsymbol{nl}\right)_{\boldsymbol{q,l}}^{(\boldsymbol{tm}-\boldsymbol{n})}}\right]\qquad\mathbf{(6)}$$

Proof. We have,

$$e_{(-m)}\left(k_{q,l}\right)=1+\frac{1}{m!}\frac{1}{k_{q,l}^{(m)}}+\frac{1}{(2m)!}\frac{1}{k_{q,l}^{(2m)}}+\frac{1}{(3m)!}\frac{1}{k_{q,l}^{(3m)}}+\cdots$$

Taking $\Delta_{q,l}^{-1}$ on both sides, we get

$$\Delta_{q,l}^{-1}\left(e_{(-m)}\left(k_{q,l}\right)\right)=\Delta_{q,l}^{-1}\left[1+\frac{1}{m!}\frac{1}{k_{q,l}^{(m)}}+\frac{1}{(2m)!}\frac{1}{k_{q,l}^{(2m)}}+\frac{1}{(3m)!}\frac{1}{k_{q,l}^{(3m)}}+\cdots\right]$$

$$\Delta_{q,l}^{-1}\left(e_{(-m)}(k_{q,l})\right) = \frac{1}{ql}\left[\frac{k_{q,l}^{(m)}}{1!} + \frac{1}{m!\,(m-1)}\frac{1}{\left(\frac{k}{q}-l\right)_{q,l}^{(m-1)}} + \frac{1}{(2m)!\,(2m-1)\left(\frac{k}{q}-l\right)_{q,l}^{(2m-1)}} + \frac{1}{(3m)!\,(3m-1)\left(\frac{k}{q}-l\right)_{q,l}^{(3m-1)}} - \cdots\right] \quad (7)$$

$$\Delta_{q,l}^{-1}\left(e_{(-m)}(k_{q,l})\right) = \frac{1}{q^l}\left[\frac{k_{q,l}^{(1)}}{1!} - \sum_{t=1}^{\infty}\frac{1}{(tm)!\,(tm-1)\left(\frac{k}{q}-l\right)_{q,l}^{(tm-1)}}\right]$$

Taking $\Delta_{q,l}^{-1}$ on both sides, in equation (7) we get,

$$\Delta_{q,l}^{-2}\left(e_{(-m)}(k_{q,l})\right) = \frac{1}{ql}\Delta_{q,l}^{-1}\left[\frac{k_{q,l}^{(1)}}{1!} - \frac{1}{m!\,(m-1)}\frac{1}{\left(\frac{k}{q}-l\right)_{q,l}^{(m-1)}} - \frac{1}{(2m)!\,(2m-1)\left(\frac{k}{q}-l\right)_{q,l}^{(2m-1)}} - \frac{1}{(3m)!\,(3m-1)\left(\frac{k}{q}-l\right)_{q,l}^{(3m-1)}} - \cdots\right]$$

Then,

$$\Delta_{q,l}^{-2}\left(e_{(-m)}(k_{q,l})\right) = \frac{1}{q^2l^2}\left[\frac{k_{q,l}^{(3)}}{3!} + \sum_{t=1}^{\infty}\frac{1}{(tm)!\,(tm-1)(tm-2)(tm-3)\left(\frac{k}{q^3}-3l\right)_{q,l}^{(tm-3)}}\right]$$

Continuing process we get,

$$\Delta_{q,l}^{-n}\left(e_{(-m)}(k_{q,l})\right) = \frac{1}{q^nl^n}\left[\frac{k_{q,l}^{(n)}}{n!}(-1)^n\sum_{t=1}^{\infty}\frac{1}{(tm)!\,(tm-1)(tm-2)\cdots(tm-n)\left(\frac{k}{q^n}-nl\right)_{q,l}^{(tm-n)}}\right]$$

Hence the proof.

□

Corollary 4.2. For $|q| \leq 1$, $|l| \leq 1$ and $k < 0$ then

$$\Delta_{q,l}^{-1}\left(e_{(-1)}\left(k_{q,l}\right)\right) = \frac{1}{ql}\left[\frac{k_{q,l}}{1!} - \sum_{t=1}^{\infty} \frac{1}{t!\,(t-1)\left(\frac{k}{q} - l\right)_{q,l}^{t}}\right]$$

Proof. Put $m = 1$ in above equation (6) we get

$$\Delta_{q,l}^{-1}\left(e_{(-1)}\left(k_{q,l}\right)\right) = \frac{1}{ql}\left[\frac{k_{q,l}}{1!} - \sum_{t=1}^{\infty} \frac{1}{t!\,(t-1)\left(\frac{k}{q} - l\right)_{q,l}^{t}}\right]$$

□

Example 4.3. Taking $n = 2, k = 4, m = 3, q = 1, l = 2$ substituting these valuesin equation (6).
In L.H.S

$$\Delta_{q,l}^{-2} e_{(3)}(4)_{1,2} = e_{(3)}(4)_{1,2} + C$$

$$= 1 + \frac{1}{3!} + \frac{1}{4_{1,2}^{(3)}} + \frac{1}{6!} + \frac{1}{4_{1,2}^{(6)}}$$

$$= 1 + 0 = 1$$

In R.H.S

$$\frac{1}{q^n l^n}\left[\frac{k_{q,l}}{1!} + (-1)^n \sum_{t=1}^{\infty} \frac{1}{(tm)!\,(tm-1)\left(\frac{k}{q^n} - nl\right)_{q,l}^{(tm-n)}}\right] = \frac{1}{1^2 . 2^2}\left[\frac{4_{1,2}^{(2)}}{2!} + 0\right]$$

$$= \frac{1}{4} \times \frac{(4)(2)}{2} + 0 = 1$$

Thus, we have verified the equation (6).

CONCLUSION:

In this paper, we have explained the higher order of mixed positive and negativeextorial function using generalized mixed difference operator. Suitable examples areprovided for verification.

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