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Application Of Graph Theory In The Success Of Managing Hospitals - A Critical Analysis

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Abstract:
Growth of hospitals is high in India. The mushroom growth of hospitals is one of the major reasons for the requirement of effective hospital administration. This research explores the utilization of equivalence relations in graph theory to address pivotal issues in hospital management, specifically focusing on the lack of infrastructure facilities, the quality of medical practitioners, and the lack of paying capacity of patients. The study aims to identify these interrelated issues by modeling them as equivalence classes within a graph structure. By analyzing the relationships and influences among these classes, the research seeks to determine the most critical issues impacting hospital management. Furthermore, the study investigates strategies to reduce the gap between these management challenges and patient satisfaction. Employing graph analysis, the research prioritizes interventions that enhance patient care and operational efficiency. The findings contribute to a comprehensive understanding of how graph theory and equivalence relations can be effectively applied to improve hospital management practices and patient outcomes.

Keywords*: Graph theory, Equivalence relation, Hospital administration, Healthcare,*

INTRODUCTION

In the complex and dynamic environment of healthcare, the relationships among medical practitioners, patients, and hospital management play a critical role in determining the efficiency and effectiveness of hospital operations (Bazzoli et al., 2004). Understanding these relationships is essential for addressing key issues such as the lack of infrastructure facilities, the lack of quality medical practitioners, and the lack of paying capacity of patients (Burroughs et al., 2000) . This study leverages the principles of graph theory and equivalence relations to analyze these relationships and propose solutions for improved hospital management and patient satisfaction (Dussault & Dubois, 2003).

Medical practitioners, including doctors, nurses, and other healthcare professionals, are the cornerstone of healthcare delivery (Gupta et al., 2024) . Their expertise, experience, and dedication directly impact patient outcomes and the overall quality of care. (Oluwaseyi Stover MSN, n.d.) However, the performance of medical practitioners is often influenced by the availability of resources, administrative

support, and professional development opportunities provided by hospital management (Hageman et al., 2015) . Ensuring that medical practitioners are well-equipped and supported is crucial for maintaining high standards of patient care (Newell & Jordan, 2015) .

Patients are the primary beneficiaries of hospital services, and their satisfaction is a key indicator of healthcare quality (Jafree et al., 2024). Patient satisfaction is influenced by various factors, including the quality of medical care, the efficiency of hospital services, and the affordability of treatment (Schoenfelder et al., 2011). Understanding the diverse needs and expectations of patients is essential for hospital management to deliver personalized and effective healthcare services (Bancsik et al., 2024). Additionally, the paying capacity of patients can affect their access to necessary treatments, making it imperative for hospitals to adopt inclusive and flexible billing practices (Honarvar et al., 2024). The management must balance the demands of maintaining infrastructure, supporting medical staff, and meeting the financial and healthcare needs of patients (Waring et al., 2016). (Rassip et al., 2023) Paying capacity of the Patients is one of the major issues in managing the hospitals.

LITERATURE REVIEW

Graph theory has emerged as a valuable tool in the field of healthcare management, offering frameworks for modeling complex relationships and dependencies within hospital systems. Hospitals are complex organizations where relationships between medical practitioners, patients, and hospital management significantly impact overall service quality and operational efficiency. By leveraging graph theory, these relationships can be analyzed systematically to address pressing issues such as resource allocation, patient satisfaction, and workforce management (Diestel, 2017).

Graph Theory in Healthcare Management

Graph theory, a branch of discrete mathematics, studies graphs, which are structures made up of nodes (or vertices) connected by edges. In the context of hospital management, nodes can represent entities like hospital administration, medical practitioners, and patients, while edges signify interactions or relationships among these entities (Gross & Yellen, 2005). This approach has proven effective in various healthcare scenarios, from managing patient flow to optimizing staff allocation. For instance, (Zhu et al., 2020) applied network analysis to identify bottlenecks in hospital systems, highlighting areas where resources were overextended and suggesting strategies to streamline patient care pathways. Similarly, (Kettinger & Grover, 1997) found that graph-based models can effectively represent interdepartmental dependencies, aiding in improving information flows and decision-making across departments

Equivalence Relations in Graph Theory

Equivalence relations in graph theory, which rely on properties such as reflexivity, symmetry, and transitivity, allow hospital administrators to categorize and analyze relationships based on shared characteristics (Diestel, 2017). These properties are particularly useful when attempting to establish priority in addressing issues like infrastructure deficiencies, quality of medical practitioners, and

patient financial capacity. By establishing equivalence relations among these factors, hospital management can classify and prioritize issues according to their influence on operational efficiency and patient satisfaction (Hanoum et al., 2022). (Dobrzykowski et al., 2015) Equivalence relations have been instrumental in similar applications, allowing administrators to classify issues, assess their relational importance, and propose targeted solutions to enhance overall healthcare delivery. Similarly (Nathan et al., 2016) using equivalence relations discussed on the lack of quality students, lack of quality faculty members and lack of proper infrastructure which lead to unemployable students.

Patient Satisfaction and Hospital Efficiency

Patient satisfaction is a primary indicator of healthcare quality, influenced by factors such as the quality of medical care, accessibility of services, and treatment affordability (Al-Abri & Al-Balushi, 2014). Studies have shown that high patient satisfaction is correlated with improved clinical outcomes and patient loyalty. For example, (Fatima et al., 2018) demonstrated that patient satisfaction increased when hospitals adopted more efficient resource allocation strategies, thus enhancing service delivery. Additionally, (Leggat & Balding, 2013) hospital management must balance clinical, administrative, and financial responsibilities to create an environment conducive to high-quality patient care and efficient operations.

Application of Graph Theory in Hospital Management

The use of graph theory in hospital management offers a structured way to understand and prioritize issues within complex healthcare systems. (Chakraborty et al., 2021) explored how network models could bridge gaps between hospital management and patient satisfaction, ultimately resulting in improved patient outcomes and streamlined operations. This approach allows administrators to identify and address inefficiencies, resource shortages, and barriers to access. Studies have confirmed that applying graph theory in such contexts enables hospitals to focus on crucial areas, ensuring that medical practitioners and other resources are deployed effectively (Wang et al., 2019). By modeling hospital systems as networks, administrators can assess the relational impact of issues, prioritize critical needs, and design interventions that align with both operational goals and patient expectations

Interconnectedness and Critical Analysis

The interconnectedness of medical practitioners, patients, and hospital management forms a complex network of dependencies and influences (Boggess et al., 2021). This study aims to dissect these relationships using graph theory and equivalence relations to identify and prioritize the most pressing issues in hospital management. By modeling these relationships, the study seeks to uncover insights into how the gaps between management challenges and patient satisfaction can be bridged, ultimately leading to enhanced healthcare delivery and improved patient outcomes.

Conceptual Framework Model

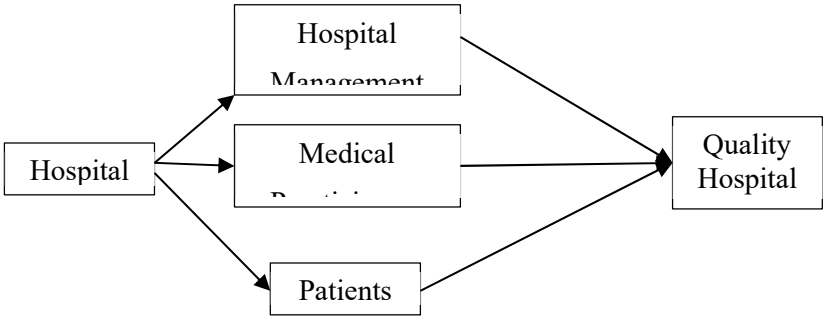

Fig 1: Conceptual Framework – Hospital

RESEARCH METHODOLOGY

Statement of the problem

Managing the Hospital has so many issues - Resource Allocation: Efficient distribution of resources affects the functionality of the hospital; Policies: Implementation of effective policies influences hospital operations; Infrastructure: Quality and maintenance of hospital facilities impact overall service delivery and Administration: Administrative efficiency is crucial for smooth operations.

When it comes to Medical Practitioners - Work Environment: A supportive and safe work environment enhances practitioner satisfaction; Professional Development: Opportunities for career growth and continuous education; Resources: Availability of necessary medical resources and equipment; Support: Administrative and peer support within the hospital and Hospitality: dealing with patients in pleasing manner.

As far as Patients are concerned: Quality of Care: High standards of medical care lead to better health outcomes and satisfaction; Accessibility: Ease of accessing healthcare services; Affordability: Reasonable costs of medical services, considering patients' paying capacity and Communication: Clear and effective communication with medical staff.

In overall, the satisfaction of the patients, lack of infrastructure facilities in hospitals and lack of quality of medical practitioners. This study also explores the major issues faced by the hospitals, patients, and medical practitioners through three different independent issues.

Objective of the study

1. To identify hospital management issues using equivalence relations in graph theory
2. To determine the most critical issues in hospital management system using graph theory
3. To bridge the gap between management issues and patient satisfaction

Metric Analysis

An equivalence relation, a powerful mathematical tool, is employed to analyze the relationships among

three critical independent issues: lack of satisfaction for the patients, lack of satisfaction for the medical practitioners, and lack of satisfaction for the hospital management.

In mathematics, let X denote a nonempty set. The equivalence relation X on X is a subset of X×X, i.e., a collection X of ordered pairs of elements of X, satisfying specific properties. By writing "x R y" to mean (x, y) is an element of X, and saying "x is related to y" the properties are defined as follows: For every a,b,c ϵ X

1. Reflexive: aRa for all a ϵ X
2. Symmetric: aRb implies bRa for all a,b ϵ X
3. Transitive: aRb and bRc imply aRc for all a,b,c ϵ X

These a,b,c ϵ X are entirely independent. (Narsingh, 1794)

Limitations

This hospital management problem covers only the three issues discussed above. There is a chance for some other issues, which were not covered in this research study.

DISCUSSION

To establish the relationships among medical practitioners, patients, and hospital management using graph theory and equivalence relations, we can model the interactions and dependencies as follows:

Graph Model

Nodes and edges play major roles in the graph model. As far as the Nodes (Vertices) are concerned Patients (P) represents individuals receiving medical care; Medical Practitioners (MP) represents doctors, nurses, and other healthcare professionals; and Hospital Management (HM) represents administrative and management staff, as well as hospital infrastructure and resources. When it comes to Edges (Connections), MP-P (Medical Practitioner to Patient) represents the direct care provided by medical practitioners to patients; MP-HM (Medical Practitioner to Hospital Management) represents the relationship between medical practitioners and hospital management, including aspects such as resource allocation, scheduling, and professional development; and P-HM (Patient to Hospital Management) represents the interaction between patients and hospital management, including factors such as patient feedback, billing, and hospital services.

Equivalence Classes:

Factors considered from Medical Practitioner Equivalence Class (EM) are education, experience, specialization, workload, and professional development. Factor considered from Patient Equivalence Class (EP) are socio-economic status, health condition, paying capacity, and satisfaction level. Factors considered for the Hospital Management Equivalence Class (EH) are infrastructure, resource allocation, administrative policies, and service quality.

Let U be the set of all successful factors related to Hospitals - U = {P, MP, HM}, where P denotes Patients, MP denotes Medical Practitioners and HM denotes Hospital Management. U should be

nonempty.

Let R be the relation on U; it is defined by R={(a,b) / a affects b}, for all a, b ϵ U

FINDINGS & ANALYSIS

Satisfaction level among medical practitioners, patients, and hospital management

To visualize the satisfaction levels among medical practitioners, patients, and hospital management, we can use a Venn diagram (Fig.2) to show the intersections of satisfaction factors among the three groups. Each section of the diagram will represent key elements that contribute to satisfaction for each group, as well as the overlapping elements that are shared among them.

Overlapping Sections

Patient Care is the prime motive expected and fulfilled between Medical Practitioners and Patients. The direct interaction where practitioners' satisfaction with their ability to provide care meets patients' satisfaction with the care received. Whereas working condition is the prime motive between Medical Practitioners and Hospital Management. Adequate resources, professional development, and support provided by hospital management influence practitioners' job satisfaction. Service Quality is the prime motive between Patients and Hospital Management. Management's role in ensuring high-quality services directly impacts patient satisfaction. Affordability, hospital policies on pricing and billing practices affect patient satisfaction.

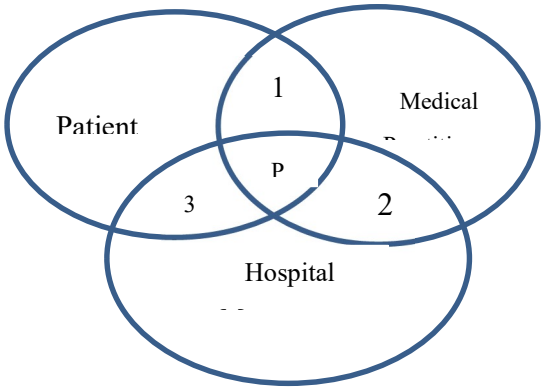

Fig2: Venn Diagram – Hospital Administration

Reflexivity:

Incapable of patients to pay attention towards his or her health (lack of cooperation and confidence among the patients in hospitals) leads to affecting themselves

Therefore patients get affected by themselves with respect to prospective hospital

P affects P

Medical practitioners not equipped themselves with respect to the updated knowledge leads to getting affected themselves

Therefore medical practitioners get affected by themselves with respect to prospective hospital

MP affects MP

Improper and outdated medical amenities in hospital with reference infrastructure will affect the hospital management

Therefore hospital management get affected by itself with respect to prospective hospital

HM affects HM

Here (P,P), (MP,MP) (HM,HM) ϵ R, for all P, MP, HM ϵ U as shown in Fig.3

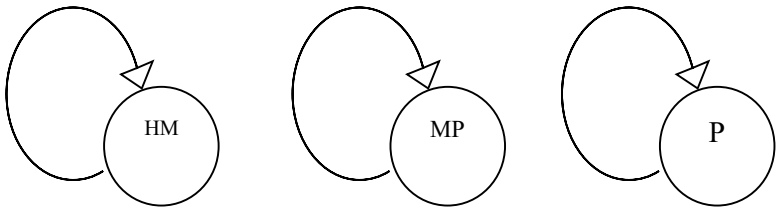

Fig.3 **Reflexive Relation**

All the issues themselves affect themselves. Therefore the above defined relation R is Reflexive.

Symmetry:

Due to the non-cooperative of the patient, the medical practitioner gets affected (spoils the reputation); similarly due to the lack of expertise of the medical practitioner in the field, the patient gets affected.

Therefore, if P affects MP then MP affects P

Lack of successful treatments of the medication practitioners to the patients affects the hospital management growth; similarly lack of hospital management support with reference to updated equipment, affects the medical practitioner performance.

Therefore, if MP affects HM then HM affects MP

Lack of more number of patients to the hospitals affects hospital management growth; similarly lack of proper updated medical equipment affects the incoming of patients in the hospital.

Therefore, if P affects HM then HM affects P

If (a,b) ϵ R, then (b,a) ϵ R, all a,b,c ϵ U as shown in Fig.4

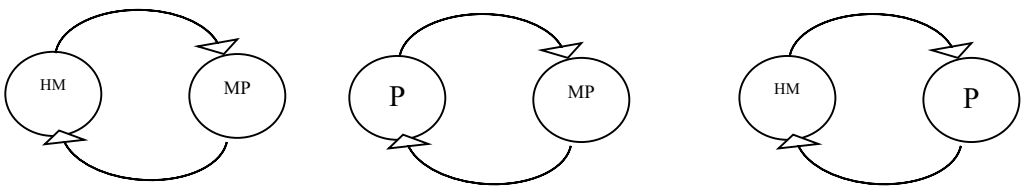

***Fig.4* Symmetry Relation**

Therefore the above defined relation R is Symmetry.

Transitivity:

Suppose non-cooperative patients affect medical practitioner (in terms of not following the instructions), and medical practitioner affects the hospital management (in terms of unsuccessful treatments), then patients affect hospital management (in terms of less patients to the hospital, which affects the growth of the hospital)

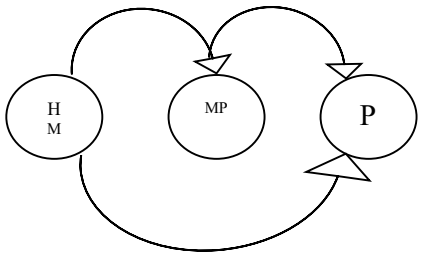

***Fig.5* Transitivity Relation**

It means if P affects MP and MP affects HM then P affects HM as shown in Fig.5

Similarly,

If (MP,P) ϵ R and (P,HM) ϵ R then (MP,HM) ϵ R

If (HM,MP) ϵ R and (MP,P) ϵ R then (HM,P) ϵ R

Therefore, if (a,b) ϵ R and (b,c) ϵ R then (a,c) ϵ R, all a,b,c ϵ U, hence the above defined relation R is transitive.

The above mentioned three issues in Hospitals satisfy the reflexive, symmetry and transitivity relations. Therefore, by definition R is an Equivalence Relation on U.

We can also justify the relationship and importance among the issues through graphical representation. Let, the vertices of the digraph be the issues of U and join vertices a & b, if and only if a affects b.

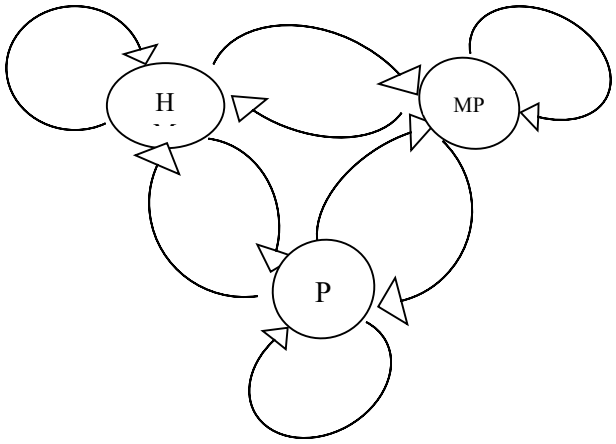

Fig.6 **Equivalence Graph**

In this graph, the equivalence classes are represented by the maximal cliques that make up the connected components. The equivalence graph exhibits strong connectivity as shown in Fig.6.

CONCLUSION

All the three independent issues of the hospital are highly dependent through the study of equivalence graph. As the graph is equally balanced, all the three issues get equal importance. All the three issues are the pillars of the hospitals with reference to quality and prospective hospital. By the method of paired comparison in preference graph (equivalence graph) the above mentioned three issues have equal ranking. No single independent issue can be avoided, as all these are nonseparable. It is suggested to develop a framework using equivalence relations in graph theory to systematically identify and categorize issues related to the management of hospitals, specifically focusing on the lack of infrastructure facilities, the quality of medical practitioners, and the paying capacity of patients. By utilizing the graph theory techniques to analyze the identified issues and determine the most critical ones among the lack of infrastructure, the quality of medical practitioners, and the paying capacity of patients. This involves assessing the impact and influence of each issue within the hospital management system. It is recommended to formulate strategies to reduce the gap between hospital management issues and patient satisfaction levels. This includes identifying key factors that affect patient satisfaction and implementing targeted interventions to address these factors, thereby enhancing overall patient care and hospital efficiency. Thus the hospitals have to concentrate in all the three issues to the maximum level to avoid the issues prevailing among the medical practitioners, patients and hospital administration.

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